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Technology Mapping for Virtual Libraries
Based on DAGs

Thesis presented in partial fulfillment of the requirements for the degree of Doctor of Computer Science

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<tr>
<td>AND</td>
<td>AND Boolean operator</td>
</tr>
<tr>
<td>BDD</td>
<td>Binary Decision Diagram</td>
</tr>
<tr>
<td>LC</td>
<td>Logic Cones</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>CSP</td>
<td>Complementary Series/Parallel</td>
</tr>
<tr>
<td>DAG</td>
<td>Directed Acyclic Graph</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programable Gate Array</td>
</tr>
<tr>
<td>LLWF</td>
<td>Library-less Wavefront</td>
</tr>
<tr>
<td>LUT</td>
<td>Lookup Table</td>
</tr>
<tr>
<td>NCSP</td>
<td>Non-Complementary Series/Parallel</td>
</tr>
<tr>
<td>NOT</td>
<td>Inversion Boolean operator</td>
</tr>
<tr>
<td>OR</td>
<td>OR Boolean operator</td>
</tr>
<tr>
<td>OTR</td>
<td>Odd-level Transistor Replacement</td>
</tr>
<tr>
<td>PC</td>
<td>Personal Computer</td>
</tr>
<tr>
<td>PD</td>
<td>Maximum number of stacked transistors in the pull-down plane</td>
</tr>
<tr>
<td>POS</td>
<td>Product of Sums</td>
</tr>
<tr>
<td>PTL</td>
<td>Pass Transistor Logic</td>
</tr>
<tr>
<td>PU</td>
<td>Maximum number of stacked transistors in the pull-up plane</td>
</tr>
<tr>
<td>ROBDD</td>
<td>Reduced Ordered Binary Decision Diagram</td>
</tr>
<tr>
<td>RTL</td>
<td>Register Transfer Language</td>
</tr>
<tr>
<td>SIS</td>
<td>Sequential Interactive System</td>
</tr>
<tr>
<td>SOP</td>
<td>Sum of Products</td>
</tr>
<tr>
<td>SPD</td>
<td>Sum of PDs in a path</td>
</tr>
<tr>
<td>SPICE</td>
<td>General-Purpose Circuit Simulation Program</td>
</tr>
<tr>
<td>SPU</td>
<td>Sum of PUs in a path</td>
</tr>
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<td>TABA</td>
<td>Tool for Library Free Technology Mapping</td>
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TSBDD  Terminal-Suppressed Binary Decision Diagram
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ABSTRACT

Technology mapping is one of the most important phases in digital circuit synthesis. This phase of logic synthesis has a major impact on the structure of the circuit, and on its delay and area characteristics. This work proposes two different approaches for library-free technology mapping aiming delay reduction in combinational circuits. The algorithms reduce the overall number of serial transistors through the longest path, considering that each transistor network of a cell has to obey to a maximum admitted chain. The number of serial transistors is computed in a Boolean way, and the mapping algorithm is performed on a Directed Acyclic Graph (DAG) description of the circuit. Preliminary results for delay were obtained through SPICE simulations. When compared to the SIS technology mapping, the proposed method shows significant delay reductions, considering circuits mapped with different libraries.

Keywords: technology mapping, logic synthesis, virtual cell library, standard cell library, automatic cell generator.
Mapeamento tecnológico para Bibliotecas Virtuais Baseado em DAGs

RESUMO

Mapeamento tecnológico é uma das principais etapas do fluxo de síntese lógica para circuitos digitais. Esta etapa define as principais características estruturais do circuito, principalmente em área e atraso. Este trabalho propõe um novo método para mapeamento de bibliotecas virtuais para redução de atraso em circuitos combinacionais. O algoritmo reduz o número de transistores em série do caminho mais longo do circuito, considerando que cada célula é implementada por uma rede de transistores que obedecem um número máximo de transistores em série. O número de transistores em série é calculado de forma booleana, garantindo que este seja o número mínimo necessário para implementar a função lógica da célula. O mapeamento é realizado sobre grafo acíclico direcionado que representa um circuito. Resultados preliminares para atraso foram obtidos por simulações SPICE. Quando comparados com o mapeamento tecnológico da ferramenta SIS, o método proposto mostra ganhos significativos em atraso, considerando circuitos mapeados com diferentes bibliotecas.

Palavras-Chave: mapeamento tecnológico, síntese lógica, bibliotecas de células, bibliotecas de células virtuais, geradores de células.
1 INTRODUCTION

One of the big challenges in high-performance circuits design is the timing closure of the control logic. Usually, control logic is not regular enough to be implemented in an intuitive design flow. Furthermore, it can be changed until the last steps of the design cycle. Therefore, Electronic Design Automation (EDA) tools are required to accelerate the design cycle. In this work, we are mainly interested in logic synthesis tools.

Logic synthesis has been shown to be an effective means of designing logic circuits, especially for semicustom designs. The computer-aided synthesis of a logic circuit involves two major steps: the optimization of a technology-independent logic representation, using Boolean and/or algebraic techniques, and technology mapping. Logic optimizations are used to modify the structure of a logic description, such that the final structure has a lower cost than the original. These optimizations are performed before the technology mapping.

Technology mapping is the step of logic synthesis that chooses the cells that will be used to implement a circuit in a given technology. This phase of logic synthesis has a major impact on the structure of the circuit, and on its delay and area characteristics. Most existing techniques are based on static pre-characterized libraries, where a set of cells is defined and characterized for a given technology. First methods for technology mapping (KEUTZER, 1987-a) (DETJENS, 1987) (ABOUZEID, 1992) (MAILHOT, 1993) (LIEN, 1992) used trees as the initial description of the circuit to be mapped. More recent methods (LEHMAN, 1997) (KUKIMOTO, 1998) (STOK, 1999) (KARANDIKAR, 2004) (MISHCHENKO, 2005) are based on Directed Acyclic Graph (DAG) representations that allow duplicating logic to some extent to increase speed. Another important contribution to technology mapping was Boolean matching (MAILHOT, 1993), where the matching of a portion of the circuit and a cell from the library is done by comparing the Boolean function of the candidates, instead of the structure. Structural comparison would not be able to find all matches.

In the early phase of technology mapping, it was considered that the use of a cell generator would enable the use of larger virtual (built on demand) cell libraries. A pioneering work aiming cell generators was presented in (BERKELAAR, 1988), which maps decomposed logic expressions onto complex gates. Another approach (REIS, 1997) uses a Binary Decision Diagrams (BDDs) representation for a circuit network, and performs BDD decomposition using constraints in the number of serial transistors. Each decomposed BDD is mapped onto a static CMOS complex gate. The work in (CORREIA, 2004) dynamically explores many embedded AND/OR decompositions by using n-ary trees for the circuit network representation. Each sub-tree that is limited by the number of serial transistors is also mapped onto static CMOS cells. In (JIANG, 2001) two techniques for technology mapping are presented. The first method maps
circuits to a virtual cell library of complex static CMOS gates. The second technique uses a mixed logic of static CMOS and PTL gates, considering the relation between PTL and BDDs.

Unfortunately, the use of such approaches was not widely verified in a commercial level, even if other references suggest that the increased number of cells in a library could lead to significant improvements in the quality of the final design (KEUTZER, 1987-b) (SCOTT, 1994) (SECHEN, 1996) (GAVRILIOV, 1997). A recent approach presented in (ROY, 2005) suggests that the addition of some custom cells to a library can improve the speed of the final circuit. Recently, some methods for generating efficient cell networks were proposed (KANECKO, 1997) (POLI, 2003) (TANAKA, 2004) (SCHNEIDER, 2005), including a method (SCHNEIDER, 2005) to compute the minimum number of transistors in series needed to implement an arbitrary Boolean function. These improvements were presented only at the cell level, lacking of an efficient method for mapping a larger circuit.

1.1 Thesis contributions

This thesis presents two different methods for “VIRtual library technology MApping” (VIRMA) based on DAGs. Both mapping methods combine the method for Boolean computation of the number of series transistors presented in (SCHNEIDER, 2005) with state of the art technology mapping algorithms inspired by the approaches presented in (STOK, 1999) and (MISHCHENKO, 2005). Significant gains are obtained in delay due to both aspects combined into the proposed mapping tool. The algorithm is library-free, as it chooses the transistor configuration for the cells that will have to be created through a cell generation tool in a subsequent step. Currently, the VIRMA mapping flow is integrated with the cell generator proposed in Rosa (2006).

1.2 Thesis organization

The remaining of this thesis is organized as follows: chapter 2 review some general concepts required for a better comprehension of the proposed methods. Specific concepts of technology mapping as well as some previous approaches are described in the chapter 3. The new approaches for library-free technology mapping are presented in the chapter 4. All results obtained through the implemented prototypes are shown in the chapter 5. Finally, conclusions are presented in the chapter 6.
2 TERMINOLOGY AND BASIC CONCEPTS

Electronic design automation is divided into many sub-areas. We are particularly interested in logic synthesis and physical synthesis (related to cell generators) concepts. This chapter introduces some of these concepts and definitions that will be used in following chapters. From now on, we assume the knowledge of all definitions described here.

2.1 Boolean logic and logic expressions

The **Boolean domain** \( B \) is defined as a two element set, say, \( B = \{0, 1\} \), whose elements are interpreted as logical values, typically \( 0 = false \) and \( 1 = true \). A Boolean function describes how to determine a Boolean value output based on some logical calculation from Boolean inputs. A **finitely Boolean function** is a function of the form \( f: B^n \rightarrow B \), where \( B = \{0, 1\} \) is a Boolean domain and where \( n \) is a non-negative integer. In the case where \( n = 0 \), the "function" is simply a constant element of \( B \). More generally, a function of the form \( f: X \rightarrow B \), where \( X \) is an arbitrary set, is a Boolean-valued function. If \( X = M = \{1, 2, 3, \ldots\} \), then \( f \) is a binary sequence, that is, an infinite sequence of 0's and 1's. If \( X = [n] = \{1, 2, 3, \ldots, n\} \), then \( f \) is a binary sequence of length \( n \). Therefore, there are \( 2^n \) such functions.

The **support** of a Boolean function is the set of variables that may change the output value of a function. For instance, consider the function \( f(a,b,c)=ab+a'c \). Its support is the set \( \{a, b, c\} \). An **input vector** is an element defined in Boolean domain and indicates the value of each variable that defines the Boolean space. An input vector \( v \in B^n \) belongs to the **ON-set** of \( f \) if and only if \( f(v) = 1 \). Otherwise, if \( f(v) = 0 \), then \( v \) belongs to the **OFF-set** of \( f \). While the vectors \( v_1 = \{1,1,0\} \) and \( v_2 = \{0,1,1\} \) belong to the ON-set of the function \( f(a,b,c)=ab+a'c \), the vector \( v_3 = \{1,0,0\} \) goes into the OFF-set of \( f(a,b,c) \).

A **logic expression** (or equation) is a Boolean function representation. Each function is unique for any application \( f: B^n \rightarrow B \) in the whole Boolean space. However, a Boolean function has infinite representations. All Boolean functions can be expressed in a canonical form through **sum of products** (SOP) or **product of sums** (POS). A SOP is said canonical when all variables appear in all products. Every instance of a Boolean variable is called **literal** according to Wagner (2006). A product of literals is formally called **cube**. For instance, \( \{a, b, c\} \) is a cube interpreted like \( a.b.c \) or just like \( abc \). A **minterm** is a cube that contains all variables of the function support.

\[
f = \overline{a} \overline{b} \overline{c} d + \overline{a} b \overline{c} \overline{d} + \overline{a} b c \overline{d} + a \overline{b} c \overline{d} + a b \overline{c} d + a b c \overline{d} + a b c d
\] (2.1)
The Figure 2.1 shows a truth table and the minterms of the Boolean function \( f \). The Equation 2.1 represents \( f \) through a SOP in a canonical form. This equation has 32 literals and it is not a minimal SOP. Karnaugh maps (KARNAUGH, 1953) and Quine-McCluskey methods that comes from (QUINE, 1955) and (MCCLUSKEY, 1956) are the principal exhaustive search techniques for two-level minimization. Although they are typically not practical algorithms, they are easy to use and simple to understand. The Espresso algorithm (McGeer, 1993) is a heuristic method for two-level minimization that is computationally less expensive and presents good results. An example of two-level minimization can be seen in the Figure 2.2. It shows the Karnaugh map for the Boolean function \( f \). The minimal cover for the ON-set is composed by four cubes. It can be represented through the Equation 2.2. The Equation 2.3 shows the minimal cover for the OFF-set of the function \( f \).
Both equations can also be represented as factored forms. According to Brayton (1987), a factored form can be defined as a representation of a logic function that is either a single literal or a sum or product of factored forms. It is very similar to a parenthesized algebraic expression. This parenthesized representation seems to be the most appropriate representation for use in multilevel logic synthesis. As an example, consider the representations in the Figure 2.3. The parenthesized expression can be seen as a logical operator tree. Any representation with more than two levels is called a multilevel representation. In this example, the logical operator tree has depth four (or four levels).

There are several methods for obtaining different factored forms for a given logic function. These methods range from purely algebraic ones, which are quite fast, to so-called Boolean ones, which are slower but are capable of giving better results. Since obtaining an optimal (shortest length) factorization for an arbitrary Boolean function is an NP-hard problem, all practical algorithms for factoring are heuristic and provide a correct, logically equivalent formula, but not necessarily a minimal length solution. A heuristic for algebraic factorization was presented in (BRAYTON, 1987). More recently, another heuristic was introduced in (MINTZ, 2005). Usually, it gets significantly better factorizations than algebraic factoring and are quite competitive with Boolean factoring but with lower computation costs.

The next section demonstrates how digital cells are constructed following some topologies and why the factored forms are important for some of them.

\[
ON = \text{set}(f) = \overline{a.c.d} + \overline{a.b.c} + a.\overline{c}.d + a.b.c \quad (2.2)
\]
\[
OFF = \text{set}(f) = \overline{a.c.d} + \overline{a.b.c} + a.\overline{c}.d + a.b.c \quad (2.3)
\]
2.2 Logic stiles of transistor networks

A logic gate or logic cell can be defined as a collection of transistors (or other electronic/electromagnetic components). The association of these components is used to implement arbitrary Boolean functions. Usually, logic gates are created following a given topology. The most common logic styles are the Pass-Transistor Logic (PTL) and the Complementary Series/Parallel (CSP) CMOS (also known as static CMOS). Independently of the topology, the output of the cell is either connected to VDD or GND through one or more paths of interconnected transistors. A path from VDD to the cell output is called pull-up path, while a path from GND to the cell output is called pull-down path.

There are several techniques for an automated transistor network generation. Buch (1997), Hsiao (2000), Shelar (2001), Shelar (2002) and Avci (2003) have proposed methods for PTL network generation. Most of them are based on Binary Decision Diagrams (BDDs). Every node of a BDD is a decision point which matches to a two-input multiplexer (refer to Figure 2.4). This element implements the Shannon expansion that is expressed by the Equation 2.4. In our BDD representation, the dashed arcs represent the decision for the negative cofactor \( f(a=0) \), while the other arcs correspond to the positive cofactor \( f(a=1) \). The composition of the cofactors and the decision variable \( a \) in this case are able to express the function \( f \) through the Shannon expansion. Due to this, a PTL cell can be easily derived from a BDD using pass transistors to build multiplexers. It will be demonstrated using the Example 1.

\[
f = a.f(a = 1) + \overline{a}.f(a = 0) \quad (2.4)
\]

![Figure 2.4: Correspondence among BDD nodes and multiplexers.](image)

**Example 1**: Consider the function \( f(a,b,c,d) \) which the ON-set is represented by Equation 2.5. Its OFF-set minimal cover is the Equation 2.6.

\[
ON - set(f) = \overline{a}\overline{b} + \overline{a}c + \overline{b}c + \overline{b}d \quad (2.5)
\]

\[
OFF - set(f) = ab + bc + acd \quad (2.6)
\]

---

1 Binary Decision Diagrams were introduced by Lee (1959). The basic idea from which this data structure was created is the Shannon decomposition (SHANNON, 1938). These two concepts resulted in efficient data structure proposed by Bryant (1986). Few years later, Brace (1990) extended it to a strongly canonical form that is so-called Reduced Ordered Binary Decision Diagram (ROBDD). In popular usage, the term BDD almost always refers to ROBDDs.
Figure 2.5: PTL transistor network derived from a BDD.

The Figure 2.5 shows a BDD that represents the Boolean function $f(a,b,c,d)$ and a possible PTL implementation. Every BDD node was replaced by a pair of transistors implementing a multiplexer that is controlled by the node variable. The resultant transistor network is only composed by NMOS transistors. Moreover, it has an inverter in the cell output which works like a signal amplifier. This way, the assignments for the source nodes (the terminal nodes 0 and 1 of the BDD) are also inverted for a correct implementation. Consider the input vector $v(a,b,c) = \{0, 1, 0\}$. In the Figure 2.6, this vector activates a path in the BDD that leads to the terminal node 1. In the PTL transistor network, a pull-down path is sensitized. Therefore, it gives $VDD$ in the inverter output which corresponds to the logical value 1.

Generally, the number of transistors in PTL cells is linearly proportional to the number of BDD nodes. There are techniques to reduce this number in few special cases. Some of these techniques are discussed in (ROSA, 2005). In PTL cells, the same intermediate node can be either in a pull-up path or in a pull-down path. When the transistor network has these sharing nodes, it is said a non-disjoint network. Another important point is the length of the longest path. The maximum number of stacked transistors corresponds to the number of variables in support of the function.

Complementary Series/Parallel cells are implemented using two disjoint transistor planes. The pull-up plane (pull-down plane) corresponds to the set of interconnected transistors between the cell output and $VDD$ (GND). While the pull-up plane is only composed by PMOS transistors, the pull-down plane is composed by NMOS transistors. Given that the pull-up plane is derived from the ON-set equation of a Boolean function, the topology of the pull-down plane is the complementary logic of the pull-up plane. In the same way, CSP cells can be derived from the OFF-set equation. The Figure 2.7 shows CSP cells derived from the ON-set and OFF-set equations (Equations 2.5 and 2.6, respectively). In the cell of the Figure 2.7a, each pull-up path matches to a cube of the Equation 2.5. The pull-down paths depends on the series/parallel associations of the pull-up plane. While the longest pull-up path has two series transistors, the longest pull-down path has four series transistors. The longest pull-up and pull-down paths of the cell derived from the Equation 2.6 have three series transistors.
2.3 Minimal length for transistor stacks in standard cell libraries

For approaches that are intended for cell generators, the number of serial transistors inside a cell is an important parameter. The methods presented in (BERKELAAR, 1988) (GAVRILOV, 1997) (REIS, 1997) (CORREIA, 2004) (JIANG, 2005) use a maximum allowed number of transistors in series as a parameter to restrict the size of feasible cells. All these approaches are limited to the use of serial/parallel implementations, and the computation of the number of serially connected transistors is done by serial/parallel association. The work in (SCHNEIDER, 2005) proposes a design methodology to implement complex gates with the exact lower bound for the number of serial transistors. More details of this approach can be also found in (SCHNEIDER 2006). It is based on the observation that the number of literals on the smallest cube of
the ON-set (OFF-set) prime irredundant cover determines the maximum number of serial transistors of the pull-up (pull-down) plane. Due to their topologies, PTL and CSP CMOS cells do not respect the lower bound in several practical cases. A different topology can be used to implement complex gates with the exact lower bound for the number serial transistors. Such topology is called Non-Complementary Series/Parallel (NCSP), where the minimum cover for the ON-set is used to derive the pull-up plane, and the minimum cover for the OFF-set is used to derive the pull-down plane. These cells do not have topological complementary plans. However, they are logically complementary.

Consider the CSP CMOS cells in the Figure 2.7. The NCSP cell which implements the Boolean function $f$ is shown in the figure 2.8. As the CSP CMOS cell, the longest pull-down path has three serial transistors. However, the smallest cube of the minimum cover for the ON-set has two literals. Thus, two serial transistors is the length of the longest pull-up path.

Accordingly to Weste (1994), the usual CSP CMOS has one important characteristic: low static power consumption. Significant power is only drawn when the transistors in the CMOS device are switching between on and off states. Although the pull-up and pull-down networks are not complementary in NCSP, they are logically complementary. Hence, there will not be any viable path connecting $VDD$ and $GND$ for any input vector. Therefore, the same principles of the CSP CMOS logic are applied to the NCSP logic. The Figure 2.9 shows this property through an example. The truth table of the function $f$ was associated to two columns named $VDD$ and $GND$. They represent the occurrence of the voltages corresponding to $VDD$ and $GND$ in the cell output.

![Figure 2.8: NCSP cell for the Boolean function $f$.](image)
2.4 Digital circuits representation

Digital circuits can be represented in many ways. Graphs\(^2\) are a widely used for this purpose. A Directed Acyclic Graph (DAG) is one of the most popular representation for logical circuits. On these structures, each logic gate is represented by a vertex and its connections by edges.

A gate \(g\) is fanin of \(g_2\) if the output of \(g\) is connected to a \(g_2\) input. This way, \(g_2\) is the fanout of \(g\). The fanout degree of a logic gate is determined by the number of logic gates connected to its output. For instance, if the logic gate \(g\) is connected to inputs of the gates \(g_2\) and \(g_3\) then its fanout degree (or just fanout) is two. When there is no gate with fanout greater than one, the circuit is called fanout-free. Fanout-free regions are also known as logic cones.

A path in a graph is an alternate sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. Formally, it is a set of vertex and connections \(\{c_0, g_0, c_1, ..., c_m, g_m, c_{n+1}\}\), where a connection \(c_i\), \(1 \leq i \leq n\), connects the output of \(g_{i-1}\) to an input of \(g_i\). Each logic gate \(g\) has a delay \(d(g)\) as well as each connection has an associated delay \(d(c)\). The delay of path is defined as \(d(P) = \sum_{i=0}^{n} d(g_i) + \sum_{i=0}^{n+1} d(c_i)\).

\(^2\) In mathematics and computer science graphs are used to model pairwise relations between objects from a certain collection. A graph refers to a collection of vertices or 'nodes' and a collection of edges that connect pairs of vertices. Each vertex has a least an incoming edge and a outgoing edge.
The depth of a graph (circuit) is the maximum number of vertexes (gates) in any path of the graph. Consequently, each vertex has a depth value that is given by the distance from the vertexes.

Consider the combinational circuit of the Figure 2.10.a. This circuit can be represented by the DAG of the Figure 2.7.b. The graph is directed from the primary outputs to the primary inputs. The logic depths, which are numerically represented, can be calculated through a depth-first search algorithm. In this example, the circuit has depth three. The networks \( n1 \) and \( n2 \) are connected to gates with fanout greater than one. In the DAG, these gates are represented by the vertexes that have more than one arriving edge. The graphical representation of Figure 2.7.b is a more formal way to see the usual graphical representation of Figure 2.7.a. Thus, we will refer DAGs just by using the representation of Figure 2.7.a.

Combinational circuits can also be represented by trees. The trees are a specialized kind of graph where all vertexes have fanout one (they are fanout-free representations). The Figure 2.10.c shows a forest of logical operator trees. Each tree corresponds to a logic cone of the circuit.

![Combinational circuit](image1)

![Representation with DAGs](image2)

![Representation with trees](image3)

Figure 2.10: Combinational circuit representations
3 TECHNOLOGY MAPPING

Technology mapping is a foundation of the logic synthesis process. It is used to define the set of elements from a library that will implement a circuit in a given technology. Typically, the objective function aims the optimal use of all gates in the library to produce a circuit with critical-path delay less than a target value and minimum area. It may sound to be an interpretation of the general logic optimization problem. However, the role of technology mapping is to finish the synthesis of the circuit by performing the final gate selection from a particular library. It assumes that the technology-independent circuit has already undergone a significant Boolean/structural optimization. In general, these algorithms do not change the structure of the circuit radically, for instance, either by finding common expressions between two or more parts of the circuit or reducing the logic depths of the critical paths. They are simplified because they are constrained by the structure of the equations produced by the technology-independent optimizations. This structural dependence has been studied by Chatterjee (2005), and it is also known as structural bias.

Most existing techniques for technology mapping are based on precharacterized libraries, and can be classified into four categories: rule-based mapping (GREGORY, 1986), graph matching (KEUTZER, 1987-a), direct mapping (LEGA, 1988) and functional matching (MAILHOT, 1993). Ideally, technology mapping algorithms should be able to satisfy several goals and handle different libraries. It is a quite hard task since the cell libraries normally have a different set of cells that implements a limited set of logic functions. A library of fixed size restricts the choices for covering a given circuit. Other approaches for technology mapping propose techniques based on cell generators. Instead of having a static library, it assumes that arbitrary cells can be generated on the fly through a cell generator, increasing the matching search space.

Besides specifying the technology mapping problem, this chapter show specific concepts and techniques for technology mapping, such as data structures, technology libraries and some of the existing methods.

3.1 Cell libraries

A cell library can be defined as a finite set of logic gates that implements different Boolean functions with different drive strengths and topologies. Traditionally, the technology mapping methods rely on static precharacterized libraries aiming delay, area and power optimizations. Each cell in the library is fully characterized through many simulations using complex numerical methods. The result of this process is a set of
accurate information about the behavior of the cell, concerning timing and power consumption, and its physical area. According to Sechen (2003), the characterization cost of a library is expensive. Hence, commercial libraries are typically composed of few hundred combinational cells and sequential elements like latches and flip-flops for which highly optimized layouts have been optimized for a particular technology. The logic designers are then restricted to using these cells in their circuit designs. The Figure 3.1 shows the usual circuit design flow considering technology mapping methodologies based on libraries with a fixed size.

A very simple library is illustrated in the Figure 3.2. The cell library names are associated to their area costs, their function and their DAG representations in terms of two-input AND/OR gates and inverters. The DAG representations correspond to graph patterns used by the matching algorithm. An equivalent library description is given in the Figure 3.3. It corresponds to a subset of the lib2.genlib that is library distribute with the SIS tool (SENTOVICH, 1992).

The quality of the mapped circuits is very dependent on the richness of the library in terms of the number of implemented logic functions, drive strengths and topologies. Libraries that implements a large number of Boolean functions leads to better results when compared to sparsely populated libraries. Keutzer (1987-b) analyzed the impact of the library size. He demonstrated that a better area optimization can be achieved using large libraries. As Jiang (2005) has observed, the most recent device technologies encourage the usage of complex gates in deep-submicron circuits. It leads to better circuit performance. Nevertheless, it complicates the problem of the traditional library-based technology mapping. In order to increase the use of complex gates in the design, the number of implemented Boolean functions has to be increased. The side effect is that this number grows exponentially. Thus, the number of gates in any library of a reasonable size can only capture a small fraction of the total number of possibilities. It makes the traditional technology mapping too restrictive.

![Figure 3.1: Digital circuits design methodology.](image-url)
<table>
<thead>
<tr>
<th>Cell</th>
<th>Cost</th>
<th>Symbol</th>
<th>Primitive graph pattern</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
<td><img src="image" alt="INV Symbol" /></td>
<td><img src="image" alt="INV Graph" /></td>
</tr>
<tr>
<td>NAND2</td>
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<td><img src="image" alt="NAND2 Symbol" /></td>
<td><img src="image" alt="NAND2 Graph" /></td>
</tr>
<tr>
<td>NAND3</td>
<td>6</td>
<td><img src="image" alt="NAND3 Symbol" /></td>
<td><img src="image" alt="NAND3 Graph" /></td>
</tr>
<tr>
<td>NAND4</td>
<td>8</td>
<td><img src="image" alt="NAND4 Symbol" /></td>
<td><img src="image" alt="NAND4 Graph" /></td>
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<tr>
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<tr>
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<td><img src="image" alt="OAI22 Symbol" /></td>
<td><img src="image" alt="OAI22 Graph" /></td>
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<tr>
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<td>10</td>
<td><img src="image" alt="XOR2 Symbol" /></td>
<td><img src="image" alt="XOR2 Graph" /></td>
</tr>
</tbody>
</table>

Figure 3.2: Static library example.
There are approaches for technology mapping based on virtual/dynamic cell libraries (it is also know by library-less technology mapping). These methods assume that each cell in the library is generated on the fly by a module generator. The Figure 3.4 illustrates the logic synthesis flow of these approaches. The mapping algorithm defines the set of cells used in the circuit implementation. This set is the input for a cell generator which provides the cell layouts that are further used in the physical synthesis.
The virtual cell libraries also have a finite number of cells. This number is limited by a set of constraints that represent characteristics of the virtual cells. These constraints can impose topological restrictions such as the maximum number of inputs. For example, consider a library restricted to 2-input cells. It results in the library in the Figure 3.5.a. Usually, virtual library-based technology mapping uses the maximum number of serial transistor to restrict the pull-up and pull-down planes of each cell. An example of library limited by two transistors in series in both planes is demonstrated in
the Figure 3.5.b. In this case, the library has only seven cells. The Table 3.1 shows the size of the virtual libraries in terms of number of cells. It uses the maximum number of serial transistors as constraint to limit the library. These values were calculated considering the CMOS CSP topology. Notice that after the limit of three serial transistors the number of cells is much bigger than a normal static library. For instance, the library (4,4) implements 3,503 distinguished Boolean functions (up to four serial transistors in both planes). This number grows exponentially when increasing the number of serial transistors.

The matching and covering methods used by the traditional technology mappers cannot be applied on the virtual library since the number of patterns is far too large. A structural/Boolean matching method, which compares the Boolean function of the cells in the library with subfunctions of the circuit representation, operates by choosing a covering that is the best solution over all possible matchings. Therefore, if the number of patterns/cells is large then more comparisons will be necessary, increasing the execution time. Virtual cell libraries do not have patterns for cell representation. There are methods, for example, to calculate the number of serial transistors of a sub-graph of the circuit representation. It is enough to know if the calculated result fits in a set of constraints. Hence, there is no need of a pattern matching algorithm.

Clearly, both the quality of the mapped circuit and the CPU time are related to the size of the library. The main barrier of the virtual library approach is the dependency on a good layout generator and the lack of accurate information about the cell behavior. Due to this, the static precharacterized libraries still popular in the industry. Currently, there are layout synthesis tools able to automatically generate layout for arbitrary complex gates more accurately and efficiently. The big problem is to find good models to estimate area, timing and power on the fly. In this sense, some estimative models have been proposed such as leakage power estimation method introduced by Butzen (2007).

Table 3.1: The size of CMOS CSP cell libraries induced by the number of serial transistors.

<table>
<thead>
<tr>
<th>PU / PD</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>7</td>
<td>18</td>
<td>42</td>
<td>90</td>
<td>186</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>18</td>
<td>87</td>
<td>396</td>
<td>1,677</td>
<td>6,877</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>42</td>
<td>396</td>
<td>3,503</td>
<td>28,435</td>
<td>222,913</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>90</td>
<td>1,677</td>
<td>28,435</td>
<td>425,803</td>
<td>6,084,393</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>186</td>
<td>6,877</td>
<td>222,913</td>
<td>6,084,393</td>
<td>154,793,519</td>
</tr>
</tbody>
</table>

Source: (SECHEN, 1996).

### 3.2 Subject graphs

All technology mapping methods are performed over a circuit representation that is called subject graph. Digital circuits can be represented in many ways through different kinds of data structures. Each algorithm uses a well defined structure accordingly to the criteria that will be explored during the technology mapping process. The evolution of the technology mapping algorithms is strongly related to their different subject graphs.
The first technology mapping methods used logical operator trees to represent parts of a circuit. Several import methods found in the literature, including more recent approaches, use this kind of subject graph (KEUTZER, 1987-a) (DETJENS, 1987) (ABOUZEID, 1992) (MAILHOT, 1993) (LIEN, 1992) (ZHAI, 2001) (CORREIA, 2004). The operator trees are very simple data structures, and they are usually used to represent logic cones of a circuit. Therefore, algorithms that use these structures can find optimal solutions for each part of the represented circuit.

The Binary Decision Diagrams are also used as subject graphs in some technology mapping approaches. There are different types of BDDs, and each one of them has a particular property. The TSBDDs were used in (REIS, 1998) (REIS, 1999) to map circuits with CMOS CSP gates. Some methods, such as (YAMASHITA, 1997) and (JIANG, 2001), use ROBDDs to map circuit to PTL cells. A recent method proposed in (ROSA, 2006) describes techniques to generated gates in different topologies, including NCSP gates.

Other methods, such as (LEHMAN, 1997) (KUKIMOTO, 1998) (STOK, 1999) (MISHCHENKO, 2005), are based on DAGs, or similar data structures, that provide a full representation of the circuit. This kind of representation does not impose limits to the boundaries of the logic cones. This way, optimal solutions can have matches that cross nets with multiple fanout.

As discussed above, each technology mapping method uses a well defined subject graph. Generally, it determines the main characteristics of the algorithm and its application. The memory consuming to store the circuit and its matches will depend on the subject graph type. Therefore, the choice of the subject graph can determine the success of a technology mapping algorithm.

### 3.3 Conventional technology mapping

As mentioned in the beginning of this chapter, the technology mapping chooses a set of logic gates to implement a circuit in a given technology. This set of cells is defined aiming the minimization of an objective function, and it is not an easy task. The majority of the algorithms reduce the technology mapping problem by handling with smaller parts of the circuit. Therefore, the result can be minimal (locally), but generally it does not correspond to global minimal costs.

The conventional technology mapping can be described as a three step procedure: technology decomposition, matching phase and covering phase. The most common methods have trees as subject description. This way, an addition step is required right before the technology decomposition. The initial graph must be partitioned into trees. This section describes each technology mapping considering a tree-based approach since it is simple and easy to understand.

#### 3.3.1 Graph partitioning

In order to apply the tree-based technology mapping, the graph must be converted in a forest of trees. It can be done by breaking the graph at each multiple-fanout point. Each node with fanout greater than one becomes the root of a tree, and fanout of this node becomes a leaf-node of another tree. Thus, this technique does not duplicate nodes in the graph. An example of partitioning is demonstrated in Figure 3.6.
3.3.2 Technology decomposition

The technology decomposition translates the graph representation into a subject graph decomposed in simple logic primitives. Any complex gate representation is replaced by a set of primitives normally composed of AND, OR and NOT operators or only AND and NOT operators. The main purpose of this step is facilitating the matching algorithm task and increases the graph granularity. The graph is decomposed using the same primitives in which the patterns of the cell in the library are represented. Therefore, the matcher can compare the circuit sub-graphs to the pattern graphs. The Figure 3.7 shows an example of technology decomposition in the tree rooted by the node n1. Three different decompositions are shown. In the first, the subject graph is expressed by AND, OR and NOT operators. All OR nodes can be replaced by a set of
AND and NOT nodes in order to reduce the number of distinguished nodes. It is demonstrated in the second subject (from left to right). As a last step, another trick can be used to increase the granularity of the subject graph. Pair of inverter nodes can be added in input that are not connected to NOT nodes.

### 3.3.3 Matching phase

This step consists on establishing the initial set of candidate matches attempting to match each node of the graph against each pattern of the library. If there are \( p \) patterns in the library and \( n \) nodes in the subject graph, then this naive approach has complexity \( O(n \cdot p) \). Structural matchers look for patterns and sub-graphs that are structurally isomorphic. Some approaches reduce the tree matching problem to the string matching problem. It is possible to find all of the strings which match a given string in time proportional to the length of the longest string in the pattern set. Boolean matchers identify patterns independently of the graph structure. There are several approaches for functional matchers that usually give better results than the structural matchers. However, these methods are expensive in terms of CPU time. Most of them have a limited search space containing Boolean functions with support up to 10 variables.

The result of the matching phase is illustrated through the Figure 3.8. The subject graph of the Figure 3.8 was matched against the library of the Figure 3.2. Only the best matches for each node are bound in the graph. All cumulative costs are also shown in Figure 3.8.

![Figure 3.8: Matching generation.](image)

### 3.3.4 Covering phase

The last step of technology mapping is the covering phase. This procedure finds the optimal set of cells for the circuit implementation. Dynamic programming is a general technique for many algorithms which can be applied to the covering problem.

Consider the problem of finding a minimum area cover for a subject tree \( T \). A scalar cost is assigned to each tree pattern, and the cost for a cover is the sum of costs for each
pattern in the cover. The minimum-area cover for a tree $T$ can be derived from the minimum-area covers for every node below the root of $T$. This is the principle of optimality for tree covering and is used as follows to find an optimal cover for $T$. For every match at the root of the tree the cost of an optimal cover containing that match equals the sum of the cost of the corresponding gate and the sum of the costs of the optimal covers for the nodes which are inputs to the match. For instance, consider the possible covers in Figure 3.9. The tree in the right side of the figure represents the minimal cover.

![Figure 3.9: Calculating cost on the subject graph.](image1)

![Figure 3.10: Tree covering result.](image2)
Note that each node in the tree is visited only once through a depth-first search algorithm. It is not necessary to re-compute the optimal cover for each input of each match. Hence, the complexity of this algorithm is proportional to the number of nodes in the subject tree times the maximum number of matches at any node in the subject tree. As a result the covering algorithm has linear complexity in the size of the subject tree, and the memory requirements are also linear in the size of the subject tree. The optimal cover of each matched tree is presented in the Figure 3.10. Its total cumulative cost is 10.

3.4 Technology mapping for standard cell libraries

This section reviews some of the main methods for technology mapping based on static precharacterized cell libraries, analyzing how they handle the technology mapping problem, the subject graphs and the objective functions.

3.4.1 Keutzer (1987) – DAGON – The first technology mapping algorithm

The first technology mapping algorithm was presented in (Keutzer, 1987-a). Keutzer has found a relationship between technology mapping and programming language compiler techniques. More specifically, matching graph patterns of a technology independent circuit representation against a library of patterns, such as standard cell libraries, is similar to match graph patterns of an intermediate representation of a computer program against the patterns of an instruction set of a given machine.

![Diagram of DAGON algorithm flow](KEUTZER, 1987)

Figure 3.11: DAGON algorithm flow (KEUTZER, 1987).
The result is an algorithm for technology mapping, called DAGON, which is able to minimize area, timing or a function of both. The initial description of a circuit is represented by a DAG. This graph is partitioned into a forest of trees that represent all logic cones of the circuit. Each sub-tree of the forest is matched against the library of patterns that are equivalent to technology cells. The tree matching is made by an external tool called twig (Tjiang, 1986). This tool was generally used to construct code generator for programming language compilers. This tool needs to be fed with list of patterns that are matched against the circuit trees. The patterns list is composed by small trees in the canonical NAND/NOT form. Who is developing the patterns is responsible to provide the cost of each one of them. Therefore, given a tree to be matched, the twig tool uses theses costs to evaluate cost of candidate matches. Once the matches and their costs are bound in the circuit tree, the covering algorithm can select the patterns/cells that will cover the tree/circuit with minimal cost. The whole DAGON mapping flow is summarized by the Figure 3.11.

Keutzer gives an example of a set of five patterns for AND-OR-INVERTER (AOI) gates. These patterns can be seen in Figure 3.12. They actually describe sixty-four unique pattern instances from an AOI444 to AOI211. Many of these patterns are symmetric, thus an AOI114 is equivalent to an AOI411. A graphical representation of the set of trees described by these patterns is given in Figure 3.13. Comparing the current cell library descriptions (refer Figure 3.3) to the library of patterns, we can see that it is a very rudimental description. It can only describe a single cost value for each
pattern while the most recent descriptions are able to describe different costs, even for each cell pin.

Since it uses the tree covering approach, optimal solutions can be found in linear time. The main disadvantage of this method is the size of the library, which is very limited. Moreover, it does not achieve good results regarding timing optimizations since it is a tree based mapping.

![Diagram](image)

Figure 3.13: Geometric interpretation of Figure 3.12 (KEUTZER, 1987).

### 3.4.2 Kukimoto (1998) – DAG based technology mapping

A pioneer work for standard cell technology mapping, which effectively uses DAGs as subject descriptions, was the algorithm introduced by Kukimoto (1998). As Keutzer (1989) showed, if a subject graph is a DAG, graph covering for minimal area mapping is a NP-complete problem. When using trees as subject graphs, some methods can guarantee a minimal cover in linear time. It leads to the usual approach of technology mapping algorithms that decompose a subject DAG into trees, solve the technology mapping problem optimally for each tree and glue the results together.

Based on these assumptions, Ruddel (1989) worked on a minimal-delay technology mapping. If loading effects are completely ignored, then the minimal-delay mapping problem for subject trees can be solved optimally by dynamic programming in linear time. In early 90’s, the appearance of the Field-programmable Gate Arrays (FPGAs) brought a new technology mapping problem. LUT-based FPGAs can implement any function of $k$ inputs in a single LUT, where $k$ is a fixed constant depending on the FPGA technology. The traditional library-based technology mapping can not be applied in FPGAs because it would be necessary to create $2^n$ patterns that correspond to the
number of Boolean functions of $n$ variables. Cong (1994) introduced a technology mapping algorithm able to find timing-optimal solutions in LUT-based FPGAs in polynomial. Unlike the conventional tree mapping, it maps a circuit directly over a DAG.

The dynamic programming approach proposed in (CONG 1994) is not specifically for FPGA. Kukimoto (1998) observed that it could be easily extended to a library-based mapping method. It was the first method to show that the minimum-delay technology mapping problem for DAGs can be solved optimally in polynomial time. This method guarantees the minimal cover under a load-independent delay model. It only considers a fixed delay between each input and the output of a gate, discarding any information about load effects. The method assumes that the mapped circuit can be continuously sized to match the delay of the matches and the actual loads. It also can take into account buffer insertion methods to reduce the load of high fanout nets. As most of the DAG-based methods, the Kukimoto’s method focuses on delay optimization without any area consideration. Therefore, at each intermediate node the fastest mapping is simply created no matter how critical the node is. It leads to a certain amount of logic duplication increasing the area of the resultant circuit.

### 3.4.3 Stok (1999) – Wavefront technology mapping

The wavefront technology mapping algorithm leads to a very simple and efficient implementation that elegantly decouples pattern matching and covering but circumvents that patterns have to be stored for the entire network simultaneously. This coupled with dynamic decomposition enables trade-off of many more alternatives than in conventional mapping algorithms. The wavefront algorithm maps optimally for minimal delay on DAGs when a gain based delay model is used. It is optimal with respect to the arrival times on each path in the network.

![Wavefront algorithm](image.png)

Figure 3.14: Wavefront algorithm (STOK, 1999).
The *wavefront* was defined as a subgraph of the DAG, such that every path from input to output goes through the subgraph. The subcircuit isolated by the wavefront is bounded by the head and the tail of the wavefront. The head of the wavefront is the boundary closer to the primary outputs (POs) and the tail of the wavefront is the boundary closer to the primary inputs (PIs) of the circuit. If, in Figure 3.14, the vertical line with label 1 is the head and the line with label 0 is the tail, the subgraph containing the inverters form the wavefront. In addition to decoupling the match generation and covering problems, the wavefront allows the match generation to work only on a subcircuit, thereby minimizing the number of matches stored at any time. Also, matches are allowed to be generated and maintained dynamically, as opposed to generating all the matches for the entire circuit a-priori.

The algorithm is illustrated through the Figure 3.14 (the same example found in Stok (1999)). It supposes that the target library, which will be used to map the circuit, is composed by the cells: NOR, AOIs, XNOR and inverters. Furthermore, the wavefront algorithm presumes that the circuit is levelized from input to output. Initially, both the head and the tail of the wavefront start at level 0 (at the PIs level). The head of the wavefront advances one step and match generation and implementation is done for all nets on this level. Matches are generated using Boolean and Structural matchers. All pattern matchers have one thing in common; given (a) net(s) in the circuit, they generate a set of technology cell matches for the subcircuit driving the(se) net(s). All technology cells and their embedding in the network are shown in dotted lines.

Unlike conventional technology mapping, matches are not limited to fanout-free regions; i.e. the match generation search process performs its search across fanout. However, the subcircuit for the match generation search process is isolated by the wavefront; i.e. the match generation search process starts at a net(s) on the head of the wavefront and stops as soon as it encounters nets on the tail of the wavefront or a net on a level below the tail of the wavefront. As matches are generated for a node, they are implemented in the underlying netlist as drivers of a multi-source net. The multiple drivers on a net include the technology-independent cell and all the technology cell matches found for that node. For instance, the net connected to the PO $x$ has four technology cells plus the technology-independent cell as drivers. The head of the wavefront keeps advancing one step at a time until matches have been generated for all nets on the highest level of the circuit.

The tail does not start moving until the head has moved for a number of steps equal to the width of the wavefront. Covering for selecting the best match occurs for all nets on the tail of the wavefront. After choose the best match of a net, a cleanup operation is recursively performed in order to remove all matches deemed sub-optimal and the technology-independent cells. Therefore, the matches for the technology cells are just kept in the sub-graph of the wavefront. The tail of the wavefront keeps advancing one step at a time until all nets on the highest level of the circuit have been covered.

The Figure 3.15 demonstrates the effect of using different wave widths. Both the quality of the mapped circuit and CPU time are affected by the width of the wavefront. While the circuit in Figure 3.15.a is the timing-optimal cover, the circuit in Figure 3.15.b is not. When using a wave width of three levels, the XNOR match is not allowed. It reduces the number of generated matches/patterns reducing the CPU time.
3.4.4 Mishchenko (2005) – Technology Mapping with Boolean Matching, Supergates and Choices

The most recent academic work for library-based technology mapping was proposed in (MISHCHENKO, 2005). It relies and enhances upon several known techniques, integrated and fine tuned to work in a new way. The previous work on DAG mapping is extended, by proposing new methods for enumerating mapping choices and performing Boolean matching, which guarantees delay-optimum phase assignment at the gate boundaries. These techniques are used to reduce the mapping dependence on the initial subject graph (CHATTERJEE, 2005) (MISCHENKO, 2006).

The technology mapping proposed by Mishchenko (2005) is devoted to minimize the delay of the longest path in the mapped netlist under a load-independent delay model. As shown by Kukimoto (1998), this problem can be optimally solved using dynamic programming for DAG-covering. The key difference of the Mishchenko’s method with the conventional approaches based on DAG-covering lies in the matching step, which uses Boolean instead of structural matching. The subject graph is represented as an And-Inverter Graph (AIG). An AIG is a DAG whose nodes represent either AND gates or primary inputs. Its edges represent wires. Inverters are represented by bubbles on the edges. An example of AIG can be seen in Figure 3.16.

![Figure 3.16: An example of an AIG (MISHCHENKO, 2005).](image)

The mapping is divided into 5 steps. First, for every node, it computes all of its $k$-feasible cuts. A feasible cut of a node $N$ in the AIG is a set of nodes $\{x_i\}$ in the transitive fan-in cone of $N$ such that an arbitrary assignment of values to $x_i$ completely determines the value of $N$. A feasible cut is redundant if the value of a node in the cut is completely
determined by an assignment of values to the other nodes in the cut. A k-feasible cut is a feasible cut of size at most $k$ that is not redundant. The cut $\{N\}$ composed of node $N$ alone is always a $k$-feasible cut of node $N$ (for any $k$) and is called the trivial cut. Each $k$-cut of node represents a Boolean function of $k$ variables. In the AIG of Figure 3.16, $\{n2\}, \{n4, n5\}, \{n4, x2, x3\}, \{n5, x1, x2\}, \{x1, x2, x3\}$ are all the 3-feasible cuts of $n2$. The algorithm for computing all $k$-feasible cuts for all nodes in the AIG is performed in one pass over the nodes as shown in Figure 3.17.

Second, for every cut, it assigns a formal variable to each node in the cut and computes the function of the corresponding node in terms of these variables. (The function is computed as a bit-vector representing the truth-table.) Third, these functions are looked up in a hash table and matched with gates from the library. Fourth, in topological order, starting with arrival times of the primary inputs, the best arrival time for each node is computed by choosing the library gate with minimum delay. Finally, in reverse topological order, the best covering is chosen. (The last two steps are exactly the same as in traditional mapping.) There is an addition post-processing step, which can recover area on the non-critical paths.

The results presented in (MISHCHENKO, 2005) demonstrate that the Boolean matching technique with optimal phase assignment is a better alternative to the graph matching since it produces superior results with shorter run-time. Supergates and choices fit nicely into this framework and greatly improve the quality of mapping by mitigating structural bias. Furthermore, the intermediate networks seen during technology independent synthesis are a useful source of choices for the final mapping. However, these techniques have limitations. The exhaustive cut computation is only practical for cuts up to 6 variables. There is a heuristic method presented in (CHATTERJEE, 2006) that allows the computation of $k$-cuts up to 12 variables.

```c
void NetworkKFfeasibleCuts( Graph g, int k ) {
  for each primary output node n of g
    NodeKFfeasibleCuts( n, k )
}

set NodeKFfeasibleCuts( Node n, int k ) {
  if ( n is primary input ) return { { n } }
  if ( n is visited ) return NodeReadCutSet( n )
  mark n as visited
  set S1 = NodeKFfeasibleCuts( NodeReadChild1( n ), k )
  set S2 = NodeKFfeasibleCuts( NodeReadChild2( n ), k )
  set Result = MergeCutSets( Set1, Set2, k ) \cup { n }
  NodeWriteCutSet( n, Result )
  return Result
}

set MergeCutSets( set Set1, set Set2, int k ) {
  set Result = {}
  for each cut Cut1 in Set1
    for each cut Cut2 in Set2
      if ( | Cut1 \cup Cut2 | \leq k ) then Result = Result \cup { Cut1 \cup Cut2 }
  return Result
}

Figure 3.17: Computation of all $k$-feasible cuts (MISHCHENKO, 2005).
3.5 Technology mapping for virtual libraries

There are some technology mapping methods based on virtual libraries. Instead of having a library with a limited amount of cells, these approaches can handle a large number of cells in libraries limited by some constraints. This section review some of these techniques focusing in the objective function, the matching algorithm and the constraints used to restrict the library.

3.5.1 Berkelaar (1988) – The first technology mapping algorithm for cell generators

The algorithm presented (BERKELAAR, 1988) was a pioneering work aiming virtual libraries. It relies on the completeness of a generated library in an automatic way. As other traditional technology mapping approaches, it works over DAG-partitioned representation. The algorithm takes as input complex Boolean expressions that represent the logic cones of the circuit. All Boolean expressions are denoted by a prefix notation, and they are assumed to be nonredundant and factorized.

The expressions are represented by a specialized kind of graph as demonstrated in Figure 3.18. Each edge symbolizes a literal of the expression. A set of three different nodes connected through two edges is equivalent to an AND operation between two literals. This is the case of the edges \( j \) and \( k \) of the graph. The association of two different nodes connected through two parallel edges is equivalent to an OR operation. Either the pair of edges \( f \) and \( g \) or \( h \) and \( i \) represent a parallel association. The size of the sub-expressions is limited to reduce the number of recursive evaluations.

The method performs a top-down traversal over the graph evaluating the expressions from source to sink (giving priority for the operators with less precedence). Each sub-graph starting from any intermediate node to the sink corresponds to a sub-expression. Each sub-expression should match with some cell in the virtual library. When the optimal match is found, the nodes in the bottom of the sub-graph become intermediate variables in the sub-expression. Then this process recursively repeated until the sub-expression become single variables.

![Figure 3.18: Graph expression representation (BERKELAAR, 1988).](image-url)
The result of the mapping process is a multi-level gate network with the minimal logic depth and the lowest possible number of gates. The main problem of this approach is that the algorithm cuts the top of the three first and the bottom intermediate nodes are not mapped yet. Therefore, the algorithm does know the depth of the uncovered sub-functions. It is estimated through a heuristic instead of solving it with a dynamic programming strategy, which could result in optimal solutions.

3.5.2 Reis (1998) – TABA

A different solution for virtual library technology mapping was proposed in (REIS, 1998). The method TABA, as it was called, uses Terminal-Suppressed Binary Decision Diagrams (TSBDDs), which are a specialized kind of BDD, as subject graph. These structures have a special property. Each edge of the TSBDD corresponds to a transistor as depicted in Figure 3.19.

This method also applies the partitioning procedure over an initial DAG to divide the representation in logic cones. Each logic cone is represented by a TSBDD. The algorithm evaluates part of the diagram verifying if they respect the constraint of the virtual library. It uses the number of serial transistors for the pull-up and pull-down networks to restrict the size of the virtual library. Each sub-TSBDD that respects the constraints is considered a viable match. Therefore, this sub-set of nodes is replaced by an intermediate node in the original TSBDD. The sub-set of nodes becomes a new TSBDD that is referenced by the intermediate nodes. It is done in a hierarchical data structure that results in a multi-level circuit representation where each instance of a TSBDD corresponds to a technology cell.

Later, the author observed that there is no advantage to have TSBDDs as subject graphs. The properties of TSBDDs are kept in tree representations. However, it was one of the first methods to claim the re-ordering of the subject graph aiming better results, mainly, in terms of logic depth.

Figure 3.19: Relation among TSBDD edges and transistor networks.
3.5.3 Jiang (2001) – OTR: PTL / CMOS technology mapping

Jiang (2001) introduces two independent algorithms that can be used to map circuits with complex gates using either PTL or CMOS cells. One of the algorithms is called Odd-level Transistor Replacement (OTR). It maps circuits using static CMOS complex gates through a topological gate collapsing method. It also uses the number of serial transistors to limit the size of the virtual library. The other algorithm performs a Boolean mapping using BDDs to derive PTL cells. As discussed in the first chapter of this work, PTL cells can be easily derived from BDDs. In the rest of this section, the OTR method will be reviewed.

The OTR method is fast and simple and avoids the intractable sub-problems in technology mapping, such as matching and covering, by constructing complex gates topologically. The basic idea of the OTR method is to use the pull-down (pull-up) transistor structure from the gates at the previous level gates to replace the pull-up (pull-down) transistors of the gates at the next level. To illustrate this, consider the circuit in Figure 3.20.a consisting of gates G1 through G7. This structure has 20 transistors in all, and a transistor-level version is shown in Figure 3.20.b. During the procedure of transforming the circuit into a complex gate, we will need to generate intermediate gates shown in Figure 3.21.a for temporary use. Those intermediate gates will be transformed into a normal static CMOS gate at the end of transformation.

As shown in the figure, we will refer to the pull-down and pull-up transistor in G1 (G2) as \( a_n \) and \( a_p \) (\( b_n \) and \( b_p \)), respectively. The pull-down (pull-up) transistors in G1 and G2 are used to replace the fanout pull-up (pull-down) transistors of these gates in G5 to obtain the gate G5\textsuperscript{'}, resulting an intermediate static CMOS gate shown in Figure 3.21.a. For example, the pull-down blocks of G1 and G2 fan out to the pull-up transistors \( p1;5 \) and \( p2;5 \) in G5, respectively, and hence \( a_n \) and \( b_n \) are inserted in their place to create G5\textsuperscript{'}. Similarly, the transistors in G3 and G4 are inserted into G6 to obtain another intermediate static CMOS gate, G6\textsuperscript{'}.

We treat these intermediate gates as intermediate synthesis stages and we will eliminate them in the next step by performing the same operation, replacing the pull-down (pull-up) block of G7 consisting of transistors \( p1;7 \) and \( p2;7 \) (\( n1;7 \) and \( n2;7 \)) by the pull-up (pull-down) blocks of the intermediate gates G5\textsuperscript{'} (G6\textsuperscript{'}). Therefore, noting that for G5\textsuperscript{'}, the pull-up block consists of \( a_n \) and \( b_n \) and the pull-down block of \( a_p \) and \( b_p \), that G6\textsuperscript{'} has a pull-up block comprising transistors \( c_n \) and \( d_n \) and a pull-down block comprising \( c_p \) and \( d_p \), we perform the following operations to obtain the final collapsed gate: (1) use \( a_n \) and \( b_n \) from G5\textsuperscript{'}, to replace \( n1;7 \) of G7 (2) use \( c_n \) and \( d_n \) from G6\textsuperscript{'}, to replace \( n2;7 \) of G7 (3) use \( a_p \) and \( b_p \) from G5\textsuperscript{'}, to replace \( p1;7 \) of G7 (4) use \( c_p \) and \( d_p \) from G6\textsuperscript{'}, to replace \( p2;7 \) of G7.

The detailed illustration of the final collapsed gate is shown in Figure 3.21.b. Note that the final implementation has only 8 transistors, a transistor count reduction of 60%.

From the principle illustrated in this example, it is easy to see that if we collapse an even number of levels of gates, we will be left with an intermediate static CMOS gate, whereas if we collapse an odd number of levels, we will return to the formal CMOS complex gate structure, and therefore this technique is called the odd-level transistor replacement (OTR) method. The number of levels used on the replacement process depends on the constraints for the maximum number of serial transistors. If the length of the longest transistor chain in a given cell is smaller than the constraint, then the cell can be used as a replacement product in the forward levels.
Figure 3.20: Circuit example (JIANG, 2001).

Figure 3.21: Logic cells generated by OTR algorithm (JIANG, 2001).
3.5.4 Correia (2004) – ELIS - Technology mapping for symmetric and asymmetric virtual libraries

The approach presented in Correia (2004) also performs technology mapping over trees. It uses n-ary trees, where nodes can have multiple outgoing edges (more than two child nodes), as subject graph. These trees allows the use of a distinct covering strategy that exploits several decompositions in the same subject description, taking into account criteria that optimize the logic depth of the nodes before gathering them into complex gates.

After the DAG partitioning procedure, each tree is transformed in a n-ary tree, with multiple input AND/OR operators and leaf nodes representing the input literals. There are two basic rules for this transformation. First, the DeMorgan’s theorem is applied in the root of a tree to propagate all inverters to the leaf boundary of the tree. It is demonstrated in the Figure 3.22.a. Afterward, every adjacent node with the same operator label is grouped into a single node as illustrated in Figure 3.22.b.

The n-ary subject trees embed all possible decompositions achieved by the application of associative transformations. As stated before, the initial decomposition of the subject graph has a significant impact in the technology mapping due to the structural bias. The impact of the initial structure can be reduced when different decompositions are explored. Thus, better results can be achieved. The Figure 3.23 shows different decompositions for the tree represented by the Figure 3.23.a.

![Figure 3.22: N-ary trees transformation rules (CORREIA, 2004).](image)

![Figure 3.23: Embedded decompositions (b,c) in a n-ary trees (a).](image)
The trees are traversed through a depth-first search procedure, and each node has its depth and (s,p) costs calculated. The series (parallel) cost of an AND (OR) node is given by the sum of all the s (p) costs of its children. The parallel (series) cost of AND (OR) node is given by the highest cost among all the s (p) costs its children. Leaf-nodes have (s,p) costs equal to (1,1). At each node where the series and parallel costs comply with the specified constraints, a match is established. The covering procedure is bottom-up, following a dynamic programming strategy. As a result, each mode whose children have already been assigned precise information about (s, p) costs. The costs for a complete tree are shown in Figure 3.24a. If a node is found having the (s,p) costs in the maximum value allowed, it is marked as an ideal cut. An example of a ideal cut made at a node for a (2,2) restriction is shown in Figure 3.24b. The sub-tree rooted at this node is then cut and directly associated to a CMOS complex gate. The cut node at the original tree is replaced by a new leaf node, with associated cost (1,1) and logic depth given by the value of the root of the recently cut tree plus 1. The search then restarts at this node. If a node is found with any of its costs exceeding the limits, it is further sliced. Its set of sons is scanned in terms of costs and depths. Several clusters grouping the nodes with lower depths are considered at this time. A new node with the same operator is added as its son, and receives a set of sons that maximizes the (s,p) costs and minimizes the depth of the new node. This new node is then cut and the cluster directly associated to a CMOS complex gate. The set of nodes removed from the original tree is substituted by a single new leaf node, with associated costs (1,1) and logic depth given by the worst value of this set plus 1. The search restarts then at the node that had some sons removed. If it still has any exceeding cost, the last step is repeated. After each iteration, the costs of the remaining nodes that root subtrees recently modified are recalculated.

The main advantage of this method is that it considers dynamically several decomposition of the subject description at low or no cost and covers it optimally in a dynamic programming manner. It finds the optimal cover in linear time. A final cut and its associated implementation is shown in Figure 3.25.
will proceed the gate polarization step, where inverts are used where necessary, prioritizing the critical paths. By exploring dynamically several decompositions embedded in the same tree, our method can achieve better results than simply relying on the initial structure provided. This improvement is accomplished at a lower computational cost than barely trying exhausting all the possibilities over numerous instances of the same description. More detailed results were present in (CORREIA, 2005). Accordingly to Keutzer (1987-b), they show that rich libraries improve the quality of the mapped circuit. A comparison against the SIS technology mapping shows significant improvements in terms of delay. The final area achieved by this approach is slightly bigger than the circuits mapped by SIS.

(a) Figure 3.25: Final cover (before inverts minimization).
4 TECHNOLOGY MAPPING USING CMOS GATES WITH MINIMUM TRANSISTOR STACKS

This chapter introduces a new approach for library-free technology mapping using CMOS gates with minimum transistor stacks (the lower bound cells). The available technology mapping methods/tools are analyzed in order to verify how these techniques can handle lower bound cells. Our method for library-free technology mapping, which is called VIRMA (“VIRtual technology MApping tool”), is described in the following. It includes a description of the object function, subject graph and all supporting functions.

4.1 Previous technology mapping techniques and CMOS gates with minimum transistor stacks

The previous chapter presented a quick overview of several technology mapping techniques that are based on different subject graphs and on dynamic or static libraries. In this section, some crucial points concerning lower bound cells are discussed. Not all available technology mapping methods can take advantage of these complex gates, either due to their subject graphs or library limitations.

4.1.1 Cell instances

The exploration of lower bound benefits in the circuit design is strongly attached to the technology mapping procedure, as stated in (ROSA, 2007).

**Lemma 1**: every function that can be expressed as prime irredundant some of products (ISOP) without repeated literals will have a complementary series-parallel (CSP) implementation with minimum length transistor chains.

**Example 1**: The cell will have a regular series-parallel implementation which is $PU=2$ and $PD=3$ (2,3). This corresponds to the lower bounds for this cell/function.

**Corollary 1**: It is a necessary condition for a cell not to respect the lower bound that it has more than one transistor in the same plane controlled by the same variable (in positive or negative polarity).

**Example 2**: The cell will have a regular series-parallel implementation which is (2,3). However, the lower bounds for this function are (2,2), and a non-complementary cell is necessary to achieve the lower bounds. This corresponds to a self-dual implementation of a carry-out circuit that is used in practice.
Corollary 2: Respecting the lower bound does not depend on the cell structure itself, but also on the way the cell is instantiated in a circuit. This way, a cell library where all the cells respect the lower bound is not sufficient to have a final circuit where all the cell instances respect the lower bounds, as some instances may not respect the lower bounds if different input pins are externally (to the cell) connected to logically equivalent signals.

Example 3: Consider the cell given by the function \( c1 = x1 \cdot x2 + x3 \cdot x4 + x5 \cdot x6 \). This cell would be available in many commercial libraries under the name OA222. This cell has a regular series-parallel implementation which is \((2,3)\). Now consider two different instances of the same cell as shown in the Figure 4.1. An instance \( f1 = a \cdot b + c \cdot d + e \cdot f \) will respect the lower bounds, as the structure of the cell is \((2,3)\) and the lower bounds for this function are also \((2,3)\). An instance \( f2 = a \cdot b + b \cdot c + a \cdot c \) will not respect the lower bounds, as the structure of the cell is \((2,3)\) and the lower bounds for the implemented functions are \((2,2)\).

Corollary 3: It is not sufficient to look at the instance of a cell to determine if it respects the lower bounds, as logic equivalency among different signals may happen through syntax that is expressed externally to the cell instance, like buffer and inverter connections.

Example 4: Consider the cell given by the function \( c1 = x1 \cdot x2 + x3 \cdot x4 + x5 \cdot x6 \) and an instance \( f1 = a \cdot b + c \cdot d + e \cdot f \). The instance will potentially respect the lower bounds, as the structure of the cell is \((2,3)\) and the lower bounds for this function are also \((2,3)\). However, if some buffers instantiated in the circuit say that \( a = e \), \( b = d \) and \( c = f \), then the function is a \((2,2)\) function. In order to discover this, it is necessary to determine which signals are equivalent through chains of inverters and buffers.

As a general conclusion about cell libraries and the respect to the lower bounds, it is possible to notice the following:

1. All the cells in the *sp.genlib* libraries distributed with SIS (SENTOVICH, 1992) have structures that respect the lower bounds. This is a consequence of these cells not presenting repeated literals. However, when instantiated in a circuit, these cells may have logically equivalent signals connected to different cell inputs and as a consequence some instances may not respect the lower bounds.

2. Most cells inserted in commercial libraries have a structure that respects the lower bounds. However this does not guarantee instances respecting the lower bounds.
3. If a cell with a structure that does not respect the lower bounds is inserted in a library, none of its instances will respect the lower bounds. However, this is a situation that rarely happens.

4. Any statistics about the number of cells that respect the lower bounds should be divided into two different questions. First question is to look at cells in the library, and see if the cell structures respect the lower bounds. This question involves only the quality of cell networks in the library. Second question is to look at cell instances in a circuit. This question evaluates how the library, the circuit, and the technology mapping fit together. In order to be valid, this second analysis should also be aware of logic equivalency expressed through buffers and inverters.

4.1.2 Tree based and DAG based technology mapping techniques

Another very important point for mapping circuits using lower bound cells are the subject graphs. Some of these representations impose some structural barriers for the mapping methods (structural bias). Most techniques are based on trees and DAGs. Trees are a fanout-free representation. It means that repeated literals can only appear in sub-trees that reach the leaf-nodes (refer Example 3 of the previous section). Moreover, the depth of the trees is generally not too long for many circuits. Therefore, it reduces the number of possibilities for implementing Boolean functions with the lower bounds for serial transistors.

On DAG representations there is path re-convergence. Thus, sub-graphs starting on any node of the DAG can represent functions with repeated literals. It makes this kind of data structure more appropriate for handling lower bound cells.

4.1.3 The computation of serial transistors

All existing library-free mapping methods such as (BERKELAAR, 1988) (GAVRILOV, 1997) (REIS, 1997) (CORREIA, 2004) (JIANG, 2005) computes the number serial transistors by serial/parallel associations. This is a structural non-Boolean way of computing the number of transistor in series, and it is monotonically increasing.

The serial (parallel) association of transistors computes the number of serial (parallel) elements of an association as the sum (maximum) of the elements in series (parallel) in the association. Following the notation (serial, parallel) or just (s, p), the computation rules are resumed below:

- **AND association**: \((s, p) \text{ cost } = (\text{sum}(s_1,\ldots,s_n), \text{max}(p_1,\ldots,p_n))\).
- **OR association**: \((s, p) \text{ cost } = (\text{max}(s_1,\ldots,s_n), \text{sum}(p_1,\ldots,p_n))\).
- **Inversions**: \((s, p) \text{ cost } = (p, s) \text{ cost}\).

The serial/parallel association of two logic functions will result in a function with a higher (or equal) number of transistors in series that the functions associated. It is demonstrated through the DAG of the Figure 4.X. The association of the sub-functions \(f3\) and \(f4\) results in the sub-function \(f6\). The \((s,p)\) cost of \(f6\) is higher than \(f3\) and \(f4\). The costs are always increasing until the node that represents \(f7\).

Schneider (2005) presented a method for Boolean computation of the number of serial transistors. The main advantages of this Boolean computation are:
1) The values are minimal.
2) The values depend solely on the candidate function and are univocally defined. They do not depend on sub-functions being associated (it is a non-structural method).

The first affirmation was demonstrated in the second chapter of this thesis using examples of NCSP cells. These cells are constructed from the minimal SOP of the ON-set and OFF-set of a Boolean function. The minimal SOP is achieved through a Boolean method which is based on the Quine-McCluskey methods. The second affirmation is quite interesting since we can conclude that the Boolean computation is not monotonically increasing. It also illustrated in the Figure 4.2. While the structural computation gives a (s,p) cost of (4,2) for the sub-function \( f_6 \), the Boolean method gives (3,2), even considering that \( f_3 \) and \( f_4 \) have higher costs. It is more apparent for the function \( f_7 \) which has cost (2,2).

Based on all explanation above, it is clear that our technology mapping algorithm depends on a Boolean method for computing the number of serial transistors. The structural method is quiet simple and its complexity is linear to the number of nodes in the sub-graph. In case of the Boolean method, the complexity is very dependent of the method that searches for the minimal SOPs. It might be a bottleneck for the mapping algorithm since it is called for each match created during the matching phase. Therefore, it must perform a very quick computation.

![Figure 4.2: Computation of serial transistors.](image)

### 4.1.4 Problem formulation

The evolution of VLSI circuits is directly related to their manufacturing processes. Accordingly to the Moore’s Law, each new generation of the lithography process brings more integration, storage capacity, performance and energy efficiency for VLSI circuits. However, the design of these circuits involving new technologies is very expensive.
Many companies would prefer to invest on good EDA tools that explore specific optimizations during the circuit design process. There is a whole search space that is usually not explored in the industrial tools. Instead of change the used technology to obtain more performance, for instance, the designer could count on efficient cells in library (either virtual or static). It leads to the approach introduced in (SCHNEIDER, 2005), which shows that transistor networks can be constructed using minimal transistor chains through NCSP gates.

As stated in the previous section, there are many occurrences of cells that do not respect the lower bounds on implementing some Boolean functions. None of the previous techniques explores the use of these cells. Recent works proposed automatic techniques to construct transistor networks that respect the lower bounds with respect of the length of serial transistor chains (ROSA, 2006). In order to verify how these cells can contribute to implement circuits with better performance, a new technology mapping method is needed.

4.2 VIRMA technology mapping tool

The VIRMA technology mapping tool puts together two concepts: library free technology mapping and lower bound cells. The first prototype to evaluate our method started to be developed early in 2005. It was strongly based on the wavefront technology mapping presented by Stok (1999). Later, a new Boolean technology mapping algorithm was introduced in (MISHCHENKO, 2005). Since this algorithm has a Boolean nature and has presented good results with static libraries, a new version of VIRMA was developed on its principles to explore the potential of virtual libraries. Both methods are detailed described in the rest of this section as well as the objective function and all helper algorithms/functions needed by the VIRMA algorithms.

4.2.1 Defining the object function

Usually, a precharacterized cell library has accurate information for each designed cell. It includes delay, area, power consumption, etc. The availability of delay information can help timing analysis algorithms, providing a very accurate timing analysis for mapped circuits. Since our technology mapping tool is not based on a precharacterized library, our cost function uses topological metrics to estimate delay costs. Basically, the number of serial transistors of the longest pull-up/pull-down path is used as metric. Even though it is not accurate, the minimization of this number leads to circuits with better delay.

In our topological model, each cell has its own cost for the pull-up plane and pull-down plane. There are also costs attached to each cell output. They represent the cost for the longest path from a primary input to the output net of the cell. The pull-up cost is calculated by the sum of serial transistors in the pull-up plane (SPU) of each cell along the path. The pull-down cost (SPD) is computed similarly using the pull-down plane costs.

The circuit in the Figure 4.3 illustrates how the costs are calculated. It is composed by the gates G1, G2, G3, G4, and G5. The set of logic cells that implements the circuit is shown in Figure 4.4. A pair of numbers represents the PU/PD costs of a logic cell. For instance, the gates G1, G2, and G3 have cost 1,2, where PU is 1 and PD is 2. Similarly, a pair of numbers represents the SPU and SPD costs for each cell output. There are three driver cells connected to the multi-source net of the primary output. The
cell with the lowest SPU is considered the fastest implementation. When there is more than one cell with the same value for SPU, the cell with the lowest SPD is deemed the fastest implementation. Therefore, the gate G5 is the fastest implementation for the net of the primary output.

4.2.2 Pre-processing procedures

As most of the technology mapping methods, VIRMA needs some pre-processing procedures before starting the matching and covering phases. First, the original circuit representation must be converted to a subject graph. Our algorithm assumes that the initial circuit is decomposed in 2-Input AND/OR gates, in order to increase the granularity of the circuit, to provide more freedom for the matching algorithm in generating arbitrary complex gates. It is represented by a DAG containing only nodes with two outgoing edges. The nodes can be primary inputs or outputs, inverters, and OR and AND operators. This procedure is demonstrated through the Figure 4.5.
a) original circuit with complex gates

b) decomposed circuit

Figure 4.5: Creating the subject graph.

Second, all inverter nodes in the subject graph are removed. This technique is used in order to reduce the total number of pattern matches, given that every possible match containing an inverter is discarded. As demonstrated in the Figure 4.6, inversion flags are placed in all incoming edges (circuit nets) connected to inverter nodes. Therefore, the signal phase assignments are preserved.

The last step is the circuit depth calculation. It is done through depth-first search algorithm. The depths are annotated in the respective nodes. Just to make it clear, inverters are not count in the depth calculation since they are not represented by DAG nodes at this moment (refer Figure 4.7). The depths are needed for both matching and covering algorithms. Each iteration of these algorithms works on a set of nodes of a given level/depth.

Figure 4.6: Inverters removal and phase assignments.

Figure 4.7: Levelizing the subject graph.
4.2.3 Post-processing procedures

Since all inverters are removed before the technology mapping algorithm, a post-processing procedure may be required to assure the correct phase assignments of all circuit nets. It can be done through a very simple procedure that verifies the polarity of each net of the circuit. The algorithm checks all inversions assignments in a net either for the inputs of the net driven cells or for the output of the driver cell.

The Figure 4.8 illustrates this procedure. The net $n$ of the Figure 4.8.a is connected to the cells $X$, $Y$, $G1$, $G2$ and $G3$ (primary inputs and outputs are also represented by node of a DAG). As the output of the gate $G1$, the inputs of the cells $X$ and $G2$ have inversion assignments. Therefore, the connection of these cells is in the correct phase. Nevertheless, the cells $Y$ and $G3$ do not have inversion assignments. Thus, an inverter is needed to correct the signal phase. This process results in the circuit of the Figure 4.8.b.

![Figure 4.8: Adjusting polarities of the circuit nets.](image)

4.2.4 VIRMA wavefront technology mapping

The VIRMA wavefront (VIRMA-WF) technology mapping was derived from the wavefront algorithm presented in (STOK, 1999). The wavefront algorithm leads to a simple implementation and maps optimally for minimal delay on DAGs using a static cell library. Since our approach is based on virtual cell libraries, some small modifications were needed. However, the original characteristics such as the management of the pattern matches and the objective function model for multi-source nets were preserved.

The VIRMA-WF method is outlined in the Figure 4.9. As the original wavefront (STOK, 1999), the wavefront is defined as a subgraph of a DAG, such that every path from input to output goes through the subgraph. The subcircuit isolated by the wavefront is bounded by the head and the tail of the wavefront. The matching generation window is given by the $\text{wave\_width}$. For instance, consider the window shown in the Figure 4.10 that has width 3. The head starts at level 0 (primary input nets). It advances level-by-level and the match generation, which is outlined in Figure 4.11, is done for all nodes on the head level. After the matching generation for a given node $n$, the covering algorithm (Figure 4.12) is immediately invoked, in order to choose the best match for $n$. These steps will be repeated until the head reaches the highest
level in the circuit. Finally, considering the inversion flags in the circuit representation, inverters are inserted when it is necessary.

**Algoritmo 4.1: Algoritmo principal do método VIRMA-WF.**

**Entrada:** circuito decomposto em portas AND/OR/NOT (DAG), número máximo de transistores em série no plano de PU, número máximo de transistores em série no plano de PD, largura da janela wavefront.

**Saída:** descrição do circuito mapeado.

```plaintext
1: procedimento wavefront_mapping(dag_cir, pu_max, pd_max, largura_janela) {
2:     remove_todos_inversores(dag_cir);
3:     calcula_niveis(dag_cir);
4:     nivel_mais_alto = retorna_nivel_mais_alto(dag_cir);
5:     linha_de_frente = 1;
6:     enquanto (linha_de_frente ≤ nivel_mais_alto) {
7:         lista_de_redes = retorna_lista_de_redes_no_nivel_da_linha_de_frente(dag_cir);
8:         para_cada rede, r em lista_de_redes {
9:             // Gera todas as combinações para r, considerando um conjunto de restrições
10:            gera_combinações(dag_cir, r, pu_max, pd_max, linha_de_frente, largura_janela);
11:            // Selecciona a melhor combinação para a rede r
12:            algoritmo_de_cobertura(dag_cir, r);
13:         }
14:         incrementa linha_de_frente;
15:     }
16:     adiciona_inversores(dag_cir);
17: }
```

Figure 4.9: Main algorithm of VIRMA-WF.

Figure 4.10: Matching generation window.
Algoritmo 4.2: Geração de combinações.

1: procedimento gera_combinações(dag_cir, r, pu_max, pd_max, linha_de_frente, largura_janela) {
2:   - no dag_cir, gera todas as combinações possíveis para a rede r, no intervalo [linha_de_frente – largura_janela : linha_de_frente]; neste momento, outras restrições também podem ser utilizadas para limitar a geração de combinações;
3:   para_cada comb na lista_de_combinações {
4:     // Calcula número mínimo de transistores em série para os planos de pull-up e pull-down
5:     custo_pu = calcula_número_mínimo_pu(comb);
6:     custo_pd = calcula_número_mínimo_pu (comb);
7:     se (custo_pu <= pu_max E custo_pd <= pd_max) {
8:       - armazena comb como uma nova célula lógica;
9:       - conecta saída da nova célula a rede r;
10:      - conecta todas as entradas da nova célula a suas redes correspondentes;
11:     }
12:   }
13: }
14: }

Figure 4.11: Matching algorithm.

Algoritmo 4.3: Algoritmo de cobertura.

1: procedimento algoritmo_de_cobertura(dag_cir, r) {
2:   - calcula as somas de pull-up para todas as células que geram a rede r;
3:   - calcula as somas de pull-down para todas as células que geram a rede r;
4:   - seleciona a célula com as menores somas de pull-up e pull-down
5:   - desconecta todas as células geradoras de r, exceto a célula selecionada, e realiza uma operação de limpeza para apagar todo o suporte das células desconectadas;
6: }

Figure 4.12: Covering algorithm.

The main difference of the VIRMA-WF method to the original wavefront technology mapping algorithm is in the matching phase. While the library based algorithm uses structural patterns for library matching, the matching algorithm of the VIRMA-WF is purely Boolean. During the matching generation procedure, the lower bound for the number of serial transistors is calculated from BDDs that represent Boolean functions of each match. All matches are enforced by the inequality pull-up_cost <= pull-down_cost. When this condition is false, the inverse function is considered (refer Figure 4.13). Therefore, the inequality will be respected. Two constraints are used to validate the patterns: max_pu and max_pd. Both limit the maximum number of serial transistors for the pull-up (max_pu) and pull-down (max_pd) chains for each match of a given circuit net. Besides these constraints, another set of restrictions can be used to avoid an excessive number of matches. For instance, the number of variables and/or the number of literals can also limit pattern matches. The matchings are not limited to fanout-free regions; i.e. the match generation search process performs its search across fanout, since these nets are in the wavefront. However, it can be easily limited to fanout-free regions testing the fanout of each net in the search space. This technique can be used in order to save area.
The library-less wavefront algorithm is illustrated by the Figure 4.14. Assume the following constraint values: $\text{max}_p = 2$, $\text{max}_p = 3$ and $\text{wave_width} = 3$. The circuit to be mapped is shown in Figure 4.14. The labeled vertical dashed lines represent the head of the wavefront. All matches and their embedding in the network are shown in dashed lines. When the head reaches the level 1, the 2-Input NAND gates are added as drivers of their respective nets in the circuit, creating multi-source nets. After the covering algorithm selects the best match for the last net on level 1, the head is moved to level 2. As the $\text{wave_width}$ is equal to the highest level of the circuit, on level 2, matches are generated until the primary inputs. The covering algorithm selects the best match for a net, and also disconnects all driver cells, except the selected match, performing a cleanup operation on their exclusive inputs. The Figure 4.16.a shows the mapped circuit under these constraints. If the $\text{wave_width}$ is reduced to 2, the circuit in the Figure 4.16.b is obtained. The same circuit is achieved, assuming the following constraints: $\text{max}_p = 1$, $\text{max}_p = 2$ and $\text{wave_width} = 2$. 

Figure 4.13: Function inversion during the mapping process.

Figure 4.14: Decomposed circuit.
4.2.5 VIRMA and $k$-cuts

The most recent approach for technology mapping based on DAGs was presented in (MISHCHENKO, 2005). This method lies on a simple exhaustive procedure able to enumerate all $k$-feasible cuts where $k$ is the number of variables in a sub-function of the circuit. Each $k$-cut is matched against the library using a Boolean matcher.

This algorithm can also be extended to realize technology mapping using virtual libraries. Therefore, the VIRMA-$K$-Cuts algorithm is introduced in this section. It uses the same subject graph as the VIRMA-WF method. Before starting the technology mapping algorithm the subject are also pre-processed. After the mapping procedure, it also performs the correctness of the net phase assignments. The difference of the Mishchenko’s method and VIRMA-$K$-cuts is in the Boolean matcher. Instead of match every $k$-cut against library of patterns, the lower bounds for the number of serial transistors are calculated through a BDD-based algorithm. If the lower bounds for the pull-up and the pull-down networks respect the constraints of the virtual library, then it is considered a valid match. As an example, consider the $k$-cuts of the Figure 4.17. The whole is circuit is covered by a $k$-cut of 4 variables. This cut is Boolean equivalent to the 4-input XORs gates presented in Figure 4.18. The implementation of this gate in the
regular CMOS CSP is prohibitive since it has eight serial transistors in the longest pull-down path. However, it can be implemented with a NCSP cell that does not have more than four serial transistors in anyone of the networks.

![Figure 4.17: K-cuts example.](image)

In library-based approaches that use structural matchers, each sub-graph rooted by a given node is matched against each pattern of the library. Although the number of possible matches is directly related to the library size, typically, it binds few patterns in the root node. It results in less CPU time to compute the accumulated match costs. Even in library-based approaches that use Boolean matchers, the number of attached matches in a node is not so high. Accordingly to Mishchenko (2005), when computing all 5-feasible cuts, the number of matches attached to each varies from 20 to 30 matches in many practical cases.

Unlike the traditional library based approaches, the VIRMA-WF has to compute all structural combinations of adjacent nodes into the wavefront window. Since the lower bound cost is not monotonically increasing, it has to compute all combinations. This number grows exponentially by increasing the wave width. It is demonstrated through the Figure 4.19. In order to find the 4-input XOR match, a wavefront with width 6 must be used. This way, the algorithm will compute 1294 combinations of candidate matches. The VIRMA-$K$-Cuts method would find less than 30 candidate matches by computing all 6-feasible cuts. This is one of the main advantages of the $k$-cuts method.

The computation of all feasible $k$-cuts has limitations. It is an exhaustive search that limited to 6 variables in most of the practical cases. Chatterjee (2006) proposed another a heuristic method to compute $k$-cuts up to 12 variables. This heuristic was not yet incorporated in the VIRMA-$K$-cuts approach.
Figure 4.18: 4-input XOR gates.

Figure 4.19: Computing the number of possible structural combinations.
4.3 Final considerations

Both VIRMA-WF and VIRMA-K-Cuts methods can be extended to support other features. Since it is not restricted to fanout-free regions, the area can grow fast during the technology mapping. In order to find a good trade-off between area and delay, the algorithm can be limited to fanout-free regions that are not critical in delay. Therefore, the non-limited approach would be used only in critical regions.

In (STOK, 1999), another extension based on a two-pass algorithm was proposed to optimize other cost functions. Instead of keeping only the best match for each network, one or more other matches are kept in the first pass. In a second pass, from the primary output to the primary inputs, the slower (smaller) patterns can be chosen in the off-critical regions to minimize area. In addition, this strategy can be used to perform a mixed technology mapping using different topologies for logic designs. The works presented in (YAMASHITA, 1997) and (JIANG, 2001) show that mixed designs using PTL and static CMOS cells can have better area/delay/power than circuits realized with only one logic style. The transistor network generator proposed in (ROSA, 2006) is able to generate PTL cells, and other different topologies as well. Thus, the two-pass approach can be used to realize mixed technology mapping using PTL and NCSP cells. Following the idea of keep more than one match for each net in the circuit, extra matches can be generated considering different topologies. Hence, matches implemented by PTL cells will be available for the second pass algorithm.
5 EXPERIMENTS

This chapter presents different sets of results in order to prove the concepts proposed in this thesis. The first set of results contemplates an analysis done over the VIRMA-WF method. These results were presented in (MARQUES, 2007). A more complete analysis of the VIRMA methods is done by comparisons among the academic state-of-art tool, ABC (MISHCHNKO, 2005), and two industrial technology mapping tools.

5.1 Comparisons among SIS and VIRMA-WF

In this section, results for a sub-set of seven ISCAS’85 benchmark circuits are presented, comparing the technology mapping performed by SIS (SENTOVICH, 1992) and VIRMA-WF (MARQUES, 2007). The algorithm prototype was developed in Java. All results were generated on a PC workstation running Windows XP using an AMD Athlon 64/3200+ processor.

For this experiment, the circuits were first decomposed into inverters and 2-Input NAND/NOR gates using SIS. Next, we performed technology mapping, using SIS and our method, for all benchmark circuits. Finally, using our cell generator, NCSP and CSP CMOS transistor networks are derived for the mapped circuits produced by our method and by SIS, respectively. Results are shown in five different tables. In tables 2-6, the first columns show the name of the circuit. The labels of the following columns describe the cell libraries used during the technology mapping. The columns 33-4 and lib2 show results for the cell libraries 33-4.genlib and lib2.genlib, respectively, mapped by SIS targeting minimum delay. Since the cell libraries 33-4.genlib and lib2.genlib have cells with costs equal or lesser than 3 for the pull-up and pull-down planes, and few cells where PU or PD cost can be 4 (such as cells found in the lib2.genlib), all circuits were mapped by our method using wave_width = 4 and the constraints 3,3 and 3,4 to limit PU and PD costs. The label T (e.g. (3,3)-T and (3,4)-T) indicates that the mapping was limited to trees (fanout free regions). The label D (e.g. (3,3)-D and (3,4)-D) indicates that the mapping was allowed to duplicate logic, resulting on DAG mapping.
Table 4.1 shows the accumulated sum of series transistors on the pull-up and pull-down planes of each cell on the longest path of the circuit. The longest path was found following the same criterions of the covering algorithm, which based on SPU and SPD costs. It is noticeable that VIRMA-WF reduces the accumulated transistor chains along the longest path. In order to prove that the reduction of transistor in the pull-up and pull-down planes can reduce the circuit delay, we used SPICE simulation to estimate delay. The transistors used on the SPICE description have fixed size. Table 4.2 presents a delay comparison between the SIS technology mapping and VIRMA-WF technology mapping. The second column (33-4 (ns)) shows delay values expressed in nanoseconds for circuits mapped by SIS. The columns 3-7 show normalized values correspondent to the delay values of the second column. VIRMA-WF method provides better results than SIS results, with average delay reductions of about 27% and 33% considering virtual cell libraries restricted by the constraints 3,3 and 3,4, respectively. The technology mapping limited to tree (T) regions performed by our method also shows improvements of 13%-15% in average. These gains demonstrate the effectiveness of having Boolean matching combined with a virtual library that is able to use lower bound cells.

Area comparison, considering the number of transistors of each circuit, can be seen in Table 4.3. Due to logic duplications during the technology mapping, inherent to DAG mapping, our method can increase the area. The area penalty for using our technology mapping algorithm is 18% and 31% in average for the virtual cell libraries 3,3 and 3,4, respectively. There are cases where the average area increase is negligible. It happens
for the technology mapping limited by fanout. Although for these cases the delay gains were not maximized, a good area/delay trade-off is still achieved.

Table 4.4 shows the execution times for SIS and VIRMA-WF, given in seconds. The VIRMA-WF is clearly more time consuming than SIS. As the time values show, they are not proportional to the size of the circuit. For instance, considering the virtual cell library (3,4), the circuit c499 uses more time than the circuit c3540. However, c499 is smaller than c3540. This is mainly due to the complexity of the lower bound calculus for each match, and also to the number of generated matches during the technology mapping process.

Table 4.5 shows results for the benchmark circuit c6288. For delay and area results, the column ‘33-4’ presents, respectively, absolute values in nanoseconds and number of transistors. The following columns show the correspondent relative values. The CPU time is expressed in seconds for all libraries. Delay gains are very significant for the libraries (3,3) and (3,4), when DAG mapping is applied. However, the area penalty is high. The c6288 is a multiplier composed by regular logic blocks, and it has several regions that are not fanout-free. Therefore, best matches that cross fanout, will probably be best matches for other regions, resulting in many duplications. This area penalty can be reduced by allowing duplication of logic only for timing critical regions.

Table 4.3: Area comparisons among SIS and VIRMA-WF.

<table>
<thead>
<tr>
<th>circuit</th>
<th>SIS</th>
<th>VIRMA-WF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3,4)-T</td>
<td>(3,4)-D</td>
</tr>
<tr>
<td>c1355</td>
<td>2140</td>
<td>1,03</td>
</tr>
<tr>
<td>c1908</td>
<td>2390</td>
<td>1,01</td>
</tr>
<tr>
<td>c3540</td>
<td>4410</td>
<td>1,09</td>
</tr>
<tr>
<td>c432</td>
<td>790</td>
<td>1,13</td>
</tr>
<tr>
<td>c499</td>
<td>1484</td>
<td>1,00</td>
</tr>
<tr>
<td>c880</td>
<td>1256</td>
<td>1,00</td>
</tr>
<tr>
<td>average</td>
<td>1,05</td>
<td>1,03</td>
</tr>
</tbody>
</table>

Table 4.4: SIS and VIRMA-WF runtime.

<table>
<thead>
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<th>VIRMA-WF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3,4)-T</td>
<td>(3,4)-D</td>
</tr>
<tr>
<td>c1355</td>
<td>0,4</td>
<td>0,3</td>
</tr>
<tr>
<td>c1908</td>
<td>0,6</td>
<td>0,5</td>
</tr>
<tr>
<td>c3540</td>
<td>2,9</td>
<td>2,6</td>
</tr>
<tr>
<td>c432</td>
<td>0,2</td>
<td>0,1</td>
</tr>
<tr>
<td>c499</td>
<td>0,3</td>
<td>0,3</td>
</tr>
<tr>
<td>c880</td>
<td>0,3</td>
<td>0,2</td>
</tr>
<tr>
<td>total (s)</td>
<td>4,7</td>
<td>4,0</td>
</tr>
</tbody>
</table>
Table 4.5: C6288 circuit results.

<table>
<thead>
<tr>
<th></th>
<th>33-4</th>
<th>lib2</th>
<th>(3,3)-CL</th>
<th>(3,3)</th>
<th>(3,4)-CL</th>
<th>(3,4)</th>
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<tbody>
<tr>
<td>delay</td>
<td>10.20</td>
<td>1.02</td>
<td>1.00</td>
<td>0.57</td>
<td>1.00</td>
<td>0.63</td>
</tr>
<tr>
<td>area</td>
<td>9444</td>
<td>1.01</td>
<td>1.00</td>
<td>1.90</td>
<td>1.00</td>
<td>2.07</td>
</tr>
<tr>
<td>CPU time (s)</td>
<td>9.0</td>
<td>8.6</td>
<td>7.6</td>
<td>1028.4</td>
<td>8.0</td>
<td>1304.0</td>
</tr>
</tbody>
</table>

Table 4.6: Area comparison – Area saving heuristic.

<table>
<thead>
<tr>
<th>circuit</th>
<th>SIS</th>
<th>VIRMA-WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1355</td>
<td>2140</td>
<td>1,03</td>
</tr>
<tr>
<td>c1908</td>
<td>2390</td>
<td>1,01</td>
</tr>
<tr>
<td>c3540</td>
<td>4410</td>
<td>1,09</td>
</tr>
<tr>
<td>c432</td>
<td>790</td>
<td>1,13</td>
</tr>
<tr>
<td>c499</td>
<td>1484</td>
<td>1,00</td>
</tr>
<tr>
<td>c6288</td>
<td>9444</td>
<td>1,01</td>
</tr>
<tr>
<td>c880</td>
<td>1256</td>
<td>1,00</td>
</tr>
<tr>
<td>average</td>
<td>1,04</td>
<td>1,00</td>
</tr>
</tbody>
</table>

Table 4.7: Delay comparison – Area saving heuristic.

<table>
<thead>
<tr>
<th>circuit</th>
<th>SIS</th>
<th>VIRMA-WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1355</td>
<td>2.32E-09</td>
<td>1,07</td>
</tr>
<tr>
<td>c1908</td>
<td>2.53E-09</td>
<td>1,08</td>
</tr>
<tr>
<td>c3540</td>
<td>3.37E-09</td>
<td>1,04</td>
</tr>
<tr>
<td>c432</td>
<td>2.70E-09</td>
<td>0,88</td>
</tr>
<tr>
<td>c499</td>
<td>1.87E-09</td>
<td>0,91</td>
</tr>
<tr>
<td>c6288</td>
<td>1.02E-08</td>
<td>1,02</td>
</tr>
<tr>
<td>c880</td>
<td>2.27E-09</td>
<td>0,91</td>
</tr>
<tr>
<td>average</td>
<td>0,99</td>
<td>0,88</td>
</tr>
</tbody>
</table>

The prototype implemented to obtain the experimental results is devoted to prove our concepts. The results show considerable delay gains. Nevertheless, area results show that better area/delay trade-offs have to be found. The area increase can be better controlled allowing duplication only in critical regions of the circuit. A very naive approach to identify critical regions was described in the previous chapter. Table 4.6 demonstrates the area effect by using this strategy considering different slack values to determine criticality of paths. When the area results area compared to the estimated delay results of Table 4.7, it clearly seems to be a good strategy to circumvent the area increasing problem. It can also decrease the CPU time, since the number of matches will be reduced. Another possibility to reduce CPU time is to store pre-computed lower bounds in a hash table, to avoid repeated computations.
5.2 Comparisons among ABC and VIRMA

This section shows comparisons of results obtained from the ABC tool that implements the algorithm proposed in (MISHCHENKO, 2005) and from both versions of the VIRMA methods: VIRMA-WF and VIRMA-K-Cuts. All circuit netlists used for comparisons came from the same starting point as the previous experiment; i.e., all circuits were decomposed in 2-inputs AND/OR gates and balanced to reduce the logic depth. After this, they were mapped using ABC and VIRMA for libraries containing cells up to 4 serial transistors in both pull-up and pull-down planes.

Table 4.8 shows the accumulated sum of series transistors on the pull-up and pull-down planes of each cell on the longest path of the circuit. Both versions of VIRMA methods achieved similar results. The sum series transistors is generally much smaller in VIRMA methods than ABC method. Table 4.9 presents a delay comparison among ABC and VIRMA technology mapping methods. The second column shows delay values expressed in nanoseconds for circuits mapped by ABC. The columns 3-4 show normalized values correspondent to the delay values of the second column. VIRMA-K-Cuts method provides better results than VIRMA-WF method. However, in most of the cases, the ABC presented slightly better results than both. VIRMA gains (up to 29%) in the c6288 benchmark (that is a multiplier) are very reasonable. The VIRMA methods handle better with XOR gates. XORs up to 4 inputs can be implemented using lower bound cells with cost (4,4), while ABC does not see these cells available in the 44-6.genlib.

Table 4.10 demonstrates area comparisons in terms of number of transistors. In average the VIRMA-WF method is 19% better than ABC, while the VIRMA-K-Cuts method is 21% better. The ABC mapping results in a considerable amount of logic duplication. It is the result of the dual-rail assignments that are made to achieve better delay results. There are area recovery heuristics implemented in the ABC environment. In order to make an appropriated comparison, one iteration of the area recovery heuristic was performed. Area recovering heuristics can also be applied in the VIRMA methods as suggested in fourth chapter (algorithm extensions).

Table 4.8: Pull-up and pull-down sums in the longest path.

<table>
<thead>
<tr>
<th>circuit</th>
<th>ABC 44-6</th>
<th>VIRMA-WF (4,4)</th>
<th>VIRMA-K-Cuts (4,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1355</td>
<td>21 28</td>
<td>16 25</td>
<td>16 25</td>
</tr>
<tr>
<td>c1908</td>
<td>30 41</td>
<td>24 41</td>
<td>24 43</td>
</tr>
<tr>
<td>c3540</td>
<td>42 49</td>
<td>32 49</td>
<td>33 52</td>
</tr>
<tr>
<td>c432</td>
<td>25 38</td>
<td>19 34</td>
<td>22 36</td>
</tr>
<tr>
<td>c499</td>
<td>21 26</td>
<td>16 25</td>
<td>16 25</td>
</tr>
<tr>
<td>c6288</td>
<td>110 127</td>
<td>83 93</td>
<td>81 94</td>
</tr>
</tbody>
</table>
Table 4.11 shows the number of instances of lower bound cells in circuits mapped by VIRMA methods. The matching algorithm finds much more sub-functions with repeated literals in DAG representations. Hence, the lower bound cells occur more frequently when a DAG is used as subject graph.

Table 4.9: Delay comparisons among ABC and VIRMA.

<table>
<thead>
<tr>
<th>circuit</th>
<th>ABC</th>
<th>VIRMA-WF</th>
<th>VIRMA-K-Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>circuits</td>
<td>44-6</td>
<td>(4,4)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>c1355</td>
<td>1,54E-09</td>
<td>1,05</td>
<td>1,00</td>
</tr>
<tr>
<td>c1908</td>
<td>2,28E-09</td>
<td>1,03</td>
<td>1,00</td>
</tr>
<tr>
<td>c3540</td>
<td>2,84E-09</td>
<td>1,15</td>
<td>1,06</td>
</tr>
<tr>
<td>c432</td>
<td>1,86E-09</td>
<td>1,26</td>
<td>1,06</td>
</tr>
<tr>
<td>c499</td>
<td>1,51E-09</td>
<td>1,08</td>
<td>1,03</td>
</tr>
<tr>
<td>c6288</td>
<td>7,79E-09</td>
<td>0,74</td>
<td>0,71</td>
</tr>
<tr>
<td>average</td>
<td>1,05</td>
<td>0,98</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.10: Area comparisons among ABC and VIRMA.

<table>
<thead>
<tr>
<th>circuit</th>
<th>ABC</th>
<th>VIRMA-WF</th>
<th>VIRMA-K-Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>circuits</td>
<td>44-6</td>
<td>(4,4)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>c1355</td>
<td>3660</td>
<td>0,75</td>
<td>0,77</td>
</tr>
<tr>
<td>c1908</td>
<td>2894</td>
<td>0,74</td>
<td>0,72</td>
</tr>
<tr>
<td>c3540</td>
<td>5824</td>
<td>0,94</td>
<td>0,80</td>
</tr>
<tr>
<td>c432</td>
<td>1108</td>
<td>1,07</td>
<td>0,87</td>
</tr>
<tr>
<td>c499</td>
<td>3368</td>
<td>0,83</td>
<td>0,84</td>
</tr>
<tr>
<td>c6288</td>
<td>17256</td>
<td>0,53</td>
<td>0,73</td>
</tr>
<tr>
<td>average</td>
<td>0,81</td>
<td>0,79</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.11: Number of instances of lower bound cells.

<table>
<thead>
<tr>
<th>circuit</th>
<th>VIRMA-WF</th>
<th>VIRMA-K-Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>circuits</td>
<td>(3,3)-T</td>
<td>(3,3)-D</td>
</tr>
<tr>
<td>#cells</td>
<td>#LB cells</td>
<td>#cells</td>
</tr>
<tr>
<td>c1355</td>
<td>280</td>
<td>0</td>
</tr>
<tr>
<td>c1908</td>
<td>358</td>
<td>2</td>
</tr>
<tr>
<td>c3540</td>
<td>937</td>
<td>39</td>
</tr>
<tr>
<td>c432</td>
<td>203</td>
<td>4</td>
</tr>
<tr>
<td>c499</td>
<td>275</td>
<td>0</td>
</tr>
<tr>
<td>c6288</td>
<td>3618</td>
<td>0</td>
</tr>
</tbody>
</table>
6 CONCLUSIONS

This thesis presented two DAG-based approaches for technology mapping using virtual libraries. Both algorithms aim delay minimization in combinational circuits. The first method, which is called VIRMA-WF, was derived from the wavefront algorithm presented in (STOK, 1999). A comparison among the traditional technology mapping performed by the SIS tool and the VIRMA-WF method were presented in (MARQUES, 2007). It shows delay reductions up to 43%. For some circuits, better delay means a high area penalty. Although the VIRMA-WF algorithm uses DAGs as subject graphs, the method was easily adapted to handle trees. It was demonstrated that the mapped circuits are 14% faster in average and have a negligible area increase.

Based on the work presented in (MISHCHENKO, 2005), a new version of VIRMA was proposed later. This approach lies on a simple algorithm that is able to enumerate all $k$-feasible cuts of a graph. Each $k$-cut is matched against the library and can be used as part of an optimal cover. Both VIRMA methods were compared to the Mishchenko’s method that is available in ABC logic synthesis tool (ABC, 2006). Even limited to $k$-cuts up to 6 variables, the VIRMA-$K$-Cuts method shows better results than the VIRMA-WF method. The ABC mapping gives slight better results in terms of delay. However, the VIRMA methods can achieve much smaller circuits.

The VIRMA methods are pioneering approaches regarding the use of cells with minimal length transistor chains. The results obtained from naive implementations show the potential of these techniques. There are several opened issues that should be better explored to improve the VIRMA methods. The implicit logic duplication that occurs in DAG based approaches can be circumvented by allowing duplications only in the critical regions of the circuit. It was demonstrated through some experiments that a good area/delay trade-off can be achieved when this technique is used. Both versions of the VIRMA methods searches for delay optimal implementations. The algorithms can be extended to a two-pass algorithm that can consider other non-delay metrics to map circuits.

One of the main problems in virtual library technology mapping approaches is the lack of accurate information of the cell characteristics. Methods to estimate area, delay and power consumption could be used to solve this problem. Butzen (2007) has presented a fast algorithm to estimate leakage-current power consumption. It could be a good approach for power estimation. The other alternative is to create hash tables with the characterization data. Therefore, a regular Boolean matcher would be needed to create the keys (for instance, binary strings) for each match. This hash tables could also
store the lower bound values in terms of serial transistors for each cell match. It would reduce the mapping time since these values would not be calculated on the fly. All techniques presented in this work are just the starting point for constructing a more powerful technology mapping tool.
REFERENCES


REIS, A. Assignation Technologique sur Bibliotheques Virtuelles de Portes Complexe CMOS. 1998. 123 F. These (Doctorate in Electronique, Optronique et Systemes) – Université de Montpellier, Grenoble.


SCHNEIDER, F; RIBAS, R.; REIS, A. Exact lower bound for the number of switches in series to implement a combinational logic cell. Dissertação de mestrado, PPGC, UFRGS, 2006.


APPENDIX A - MAPEAMENTO TECNOLÓGICO PARA BIBLIOTECAS VIRTUAIS BASEADO EM DAGS

Um dos maiores desafios no projeto de circuitos integrados de alto-desempenho é atingir as metas de atraso na lógica de controle. Normalmente, a lógica de controle não é regular o suficiente para ser implementada em um fluxo de projeto intuitivo. Além disso, este tipo de lógica sofre modificações até as últimas etapas do ciclo de projeto. Consequentemente, a síntese lógica é necessária para viabilizar a implementação dos circuitos, garantindo uma implementação rápida e correta de sub-circuitos irregulares.

A maioria das ferramentas de síntese lógica consiste de três etapas bem definidas. O mapeamento tecnológico normalmente segue uma etapa otimizações em uma descrição do circuito, sendo esta independente de tecnologia. A etapa posterior ao mapeamento tecnológico é uma etapa para otimizações específicas, considerando as restrições de atraso do circuito.


Além das abordagens que realizam mapeamento tecnológico com bibliotecas estáticas, outras abordagens consideram o uso de geradores de células, o que possibilitaria o uso de uma grande biblioteca virtual (construída sob demanda). Um trabalho pioneiro visando geradores de células foi apresentado por (BERKELAAR, 1988). Neste método, expressões lógicas decompostas são mapeadas em portas complexas. Um outro método, proposto em (REIS, 1997), utiliza Diagramas de Decisão Binária (BDDs) para representar um circuito e aplica decomposições nestas estruturas, considerado uma restrição para o número máximo de transistores em série admitido para implementar uma dada função Booleana. Cada BDD decomposto é mapeado em uma porta complexa CMOS estática. O trabalho apresentado em (CORREIA, 2004) explora dinamicamente decomposições AND/OR usando árvores de grau livre que representam o circuito a ser mapeado. Cada sub-árvore é limitada pelo número de transistores em
série necessários para implementar uma célula CMOS estática. Em (JIANG, 2005) duas técnicas para mapeamento tecnológico são apresentadas. O primeiro método mapeia circuitos para uma biblioteca virtual de células CMOS estáticas. A segunda técnica usa uma lógica mista de células CMOS e PTL, considerando a relação direta de uma célula PTL e um BDD.


Esta trabalho apresenta um método para mapeamento tecnológico, que combina o método Booleano para calcular o número mínimo de transistores em série, apresentado em (SCHNEIDER, 2005), com o algoritmo para mapeamento tecnológico proposto por (STOK, 1999). Ganhos significativos em atraso são obtidos combinando estes dois aspectos. O algoritmo seleciona configurações de transistores para células que serão concebidas por um gerador automático de células (ROSA, 2006) em uma etapa posterior ao mapeamento.

O principal objetivo do método é a minimização do atraso dos circuitos mapeados. Uma comparação entre o mapeamento tecnológico tradicional da ferramenta SIS e o método proposto, utilizando diferentes bibliotecas virtuais, mostra que as reduções em atraso vão de 6% a 48%. Para alguns circuitos, um atraso menor significa um grande aumento em área. O método VIRMA limitado a cones lógicos produz circuitos com um aumento de área insignificante e com um razoável ganho médio de 14% em atraso.

Estes resultados estão publicados em Marques (2007) e revelam a necessidade de criar heurísticas que evitem um aumento excessivo na área do circuito mapeado. Espera-se obter melhores resultados impondo limites para as duplicações de lógica. Isto deve ocorrer somente em regiões críticas do circuito. Estas limitações tendem a diminuir o número de combinações possíveis durante o mapeamento. Logo, o tempo de execução do algoritmo também deve diminuir. O método proposto não possui um modelo de atraso preciso. (SUTHERLAND, 1999) propõe uma metodologia para estimativa de atraso, que é baseada no dimensionamento de transistores e parâmetros reais de um determinado processo de fabricação de circuitos. Esta metodologia pode ser utilizada tanto para dimensionar transistores quanto para estimar atraso e área de um circuito.
O método VIRMA é inovador, pois é o primeiro algoritmo para mapeamento tecnológico com bibliotecas virtuais que considera o uso de células lógicas com cadeias mínimas de transistores em série. Com as implementações das extensões do algoritmo e a metodologia de (SUTHERLAND, 1999), pretende-se obter um fluxo de síntese capaz de gerar descrições de circuitos em nível de transistores, a fim de utilizá-las em ferramentas que realizam síntese física.
How to run the VIRMA tool

`java -Xmx512m -Xms256m -jar virma.jar <parameters> [options]`

If you run it without any parameter or option, a brief help will be printed out. You do not need to specify the parameters and options in a pre-defined order. It is recommended to use 512Mb for the main memory and 256Mb for the stack memory.

Parameters description

- **Input netlist**: VIRMA can read EQN and BLIF files.
- **Output netlist**: the output netlist is a “mapped” EQN file. In this file, each equation represents a cell and it is associated to a logic style (CSP or NCSP). The logic style is used for the cell generator.
- **Constraints and the Virtual Library**: the virtual library is defined by two constraints: pu and pd. They determine the maximum number of transistors in the pull-up (pu) plane and in the pull-down (pd) plane of a cell. The parameters ‘max_tree_k_cut’ and ‘max_dag_k_cut’ define the maximum number of variables for matches generated from trees and DAGs. The implemented algorithm for finding the k-cuts is very limited. Therefore, practical values for these constraints are 10 and 6, respectively. Otherwise, it could take too long time for mapping a circuit.
- **Options**: since VIRMA maps a circuit for a virtual library, a list of cells can be provided for a cell generator. This list will contain all cells used in the mapped circuit. In order to generate it, you just need to specify the a file name through the option ‘-output_library’. If you use the option ‘-lc’ (short for library costs), the CSP and NSCP costs will be written in the output library file. The options ‘–CSP’ and ‘–NSCP’ are used to specify the logic style used for mapping. By default, the NSCP is taken. If you choose CSP, only the standard CMOS cells will be used.